M.Sc. Final Year

Physics, MP-07

NUCLEAR PHYSICS AND PARTICLE PHYSICS

मध्यप्रदेश भोज (मुक्त) विश्वविद्यालय – भोपाल

MADHYA PRADESH BHOJ (OPEN) UNIVERSITY – BHOPAL

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Units (1-5)

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SYLLABI-BOOK MAPPING TABLE

Nuclear Physics and Particle Physics

Syllabi	Mapping in Book
Unit I **Basic Properties of Nuclei:** Methods for determination of nuclear size and their interpretations (experimental details not required) Binding energy curve for nuclei and its consequences Numerical problems) Nuclear spin. Magnetic and quadrupole moments of nuclei (with experimental determination) Schmidt's lines. Semi-emprical mass formula and its application to mass parabolas. Mirror nuclei and isotopic spin formalism.	**Unit-1:** Basic Properties 0f Nuclei **(Pages 3-40)**
Unit II **Nuclear Force and Two Body Problem:** Basic properties of deuteron, its binding energy, size, spin, magnetic and quadrupole moments. Existence of excited states of deuteron (solution of spherically symmetric square well potential for higher angular momentum states), n-p scattering at low energies with specific square well potential results of low energy n-p and p-p scattering. Their spin dependence and scattering length. Qualitative discussion of the effective range theory. Theory of n-p and p-p scattering (excluding mathematical derivation. Results of the theory to be assumed). Various types of two body nuclear forces, Elementary idea of Yukawo theory of nuclear forces.	**Unit-2:** Nuclear Force and two Body Problem **(Pages 41-78)**
Unit II **Nuclear Models and Nuclear Reactions:** Liquid drop model and fission, Asymmetric in fission and spontaneous fission. Nuclear shell model. Spin orbit coupling, Magnetic and quadrupole moments and the nuclear shell structure of elementary idea of collective model of the nucleus. Conservation loss of nuclear reactions Q value and threshold energy of a nuclear reaction. Nuclear reaction cross section and level width, Bohr compound nucleus theory of nuclear reaction. Deuteron stripping reaction. Briet-Wigner single level formula.	**Unit-3:** Nuclear Models and Nuclear Reactions **(Pages 79-128)**
Unit IV **Nuclear Decay:** Decay and nature of Beta ray spectrum. Neutrino hypothesis.Fermi theory of Beta decay allowed forbidden transitions parity violation in Beta decay, Concept of helicity. Multiple transition and selection rules for the decay of the nuclei selection rules (results without mathematical derivation). Internal conversion coefficients Isomeric nuclei. Angular correlation of su ccessive decay.	**Unit-4:** Nuclear Decay **(Pages 129-154)**
Unit V: Elements of Particle Physics **Elementary Particles:** Classification and their interaction, concept of quantum number lsospin hypercharge, strangeness, Leptons and Baryon numbers. Invariance, concept variation laws and selection rules in relation to particle production and decay of charge conjugation, parity invariance and time reversal with simple application in particle physics, Elementary idea SU(2) and SU (3) Results of group theory be assumed. Gell-Mann Okubo mass formula (without derivation) and its application to mass spectra of particles qualitative idea of quark Lepton family and quantum chromodynamics.	**Unit-5:** Elements of Particle Physics **(Pages 155-186)**

CONTENTS

INTRODUCTION **1**

UNIT 1 BASIC PROPERTIES OF NUCLEI **3–40**

1.0 Introduction
1.1 Objectives
1.2 Methods for Determination of Nuclear Size and their Interpretations
1.3 Binding Energy Curve for Nuclei and Its Consequences
1.4 Nuclear Spin
1.5 Magnetic and Quadrupole Moments of Nuclei
1.5.1 Quadrupole Moments of Nuclei
1.6 Schmidt Lines
1.7 Semi-Empirical Mass Formula and Its Application to Mass Parabolas
1.8 Mirror Nuclei and Isotopic Spin Formalism
1.8.1 Basic Properties of Nuclei
1.9 Answers to 'Check Your Progress'
1.10 Summary
1.11 Key Terms
1.12 Self Assessment Questions and Exercises
1.13 Further Reading

UNIT 2 NUCLEAR FORCE AND TWO BODY PROBLEM **41–78**

2.0 Introduction
2.1 Objectives
2.2 Deuteron: Basic Properties
2.3 Existence of Excited States of Deuteron
2.4 n-p Scattering at Low Energies with Specific Square Well and Potential Results
2.5 Qualitative Discussion of the Effective Range Theory
2.6 Theory of p-p Scattering, Spin Dependence and Scattering Length
2.6.1 Spin Dependence of n-p Interaction
2.6.2 Scattering Length
2.6.3 p-p Scattering at High Energy
2.7 Various Types of Two Body Nuclear Forces
2.8 Elementary Idea of Yukawa Theory of Nuclear Forces
2.9 Answers to 'Check Your Progress'
2.10 Summary
2.11 Key Terms
2.12 Self Assessment Questions and Exercises
2.13 Further Reading

UNIT 3 NUCLEAR MODELS AND NUCLEAR REACTIONS 79–128

3.0 Introduction
3.1 Objectives
3.2 Liquid Drop Model and Fission
3.3 Bohr and Wheeler's Theory
3.3.1 Asymmetric Fission
3.3.2 Spontaneous Fission
3.4 Nuclear Shell Model
3.5 Spin Orbit Coupling
3.6 Magnetic and Quadrupole Moments
3.6.1 Quadrupole Moments
3.7 Nuclear Shell Structure
3.7.1 Elementary Idea of Collective Model of the Nucleus
3.8 Conservation Laws of Nuclear Reactions and Q Value
3.9 Threshold Energy of a Nuclear Reaction
3.10 Nuclear Reaction
3.10.1 Cross Section and Level Width
3.11 Bohr Compound Nucleus Theory of Nuclear Reaction
3.11.1 Deuteron Stripping Reaction
3.12 Breit-Wigner Single Level Formula
3.13 Answers to 'Check Your Progress'
3.14 Summary
3.15 Key Terms
3.16 Self Assessment Questions and Exercises
3.17 Further Reading

UNIT 4 NUCLEAR DECAY 129-154

4.0 Introduction
4.1 Objectives
4.2 Beta Ray Spectrum: Decay and Nature
4.3 Neutrino Hypothesis
4.4 Fermi Theory of Beta Decay: Allowed and Forbidden Transitions Parity Violations in Beta Decay
4.5 Concept of Helicity
4.6 Multipole Transition and Selection Rules for the Decay of the Nuclei Selection Rules
4.7 Internal Conversion, Conversion Coefficients of Isomeric Nuclei
4.8 Angular Correlation of Successive Decay
4.9 Answers to 'Check Your Progress'
4.10 Summary
4.11 Key Terms
4.12 Self Assessment Questions and Exercises
4.13 Further Reading

UNIT 5 ELEMENTS OF PARICLE PHYSICS 155–186

5.0 Introduction
5.1 Objectives
5.2 Classification of Elementary Particles and their Interactions
5.3 Concept of Quantum Numbers
5.4 Isospin Hypercharge
5.5 Strangeness
5.6 Lepton and Baryon Numbers
5.7 Invariance
5.8 Concept of Variation Laws and Selection Rules in Relation to Particle Production
5.9 Decay of Charge Conjugation
5.1 Parity Invariance and Time Reversal with Simple Application in Particle Physics
5.11 Elementary Idea Su(2) and Su(3) Results of Group Theory
5.12 Gell-Mann-Okubo Mass Formula (Without Derivation) and Its Application to Mass Spectra of Particles
5.13 Qualitative Idea of Quark Lepton Family and Quantum Chromodynamics
5.14 Answers to 'Check Your Progress'
5.15 Summary
5.16 Key Terms
5.17 Self Assessment Questions and Exercises
5.18 Further Reading

INTRODUCTION

NOTES

The indication that all matter is fundamentally composed of elementary particles dates from at least the 6th century BC. The early 20th century explorations of nuclear physics and quantum physics led to proofs of nuclear fission in 1939 by Lise Meitner (based on experiments by Otto Hahn), and nuclear fusion by Hans Bethe in that same year.

Particle physics, also known as high energy physics, is a branch of physics that studies the nature of the particles that constitute matter and radiation. Although the word particle can refer to various types of very small objects, such as the protons, gas particles, or even household dust, particle physics usually investigates the irreducibly smallest detectable particles and the fundamental interactions necessary to explain their behaviour. These elementary particles are excitations of the quantum fields that also govern their interactions.

Modern particle physics research is focused on subatomic particles, including atomic constituents, such as electrons, protons, and neutrons (protons and neutrons are composite particles called baryons, made of quarks), produced by radioactive and scattering processes, such as photons, neutrinos, and muons, as well as a wide range of exotic particles. Dynamics of particles is also governed through the quantum mechanics since they exhibit wave–particle duality, displaying particle-like behaviour under certain experimental conditions and wave-like behaviour in others. In more technical terms, they are described by quantum state vectors in a Hilbert space, which is also treated in quantum field theory.

This book, *Nuclear Physics and Particle Physics* is divided into five units that follow the self-instruction mode with each unit beginning with an Introduction to the unit, followed by an outline of the Objectives. The detailed content is then presented in a simple but structured manner interspersed with Check Your Progress Questions to test the student's understanding of the topic. A Summary along with a list of Key Terms and a set of Self-Assessment Questions and Exercises is also provided at the end of each unit for recapitulation.

UNIT 1 BASIC PROPERTIES OF NUCLEI

NOTES

Structure

1.0 Introduction
1.1 Objectives
1.2 Methods for Determination of Nuclear Size and their Interpretations
1.3 Binding Energy Curve for Nuclei and Its Consequences
1.4 Nuclear Spin
1.5 Magnetic and Quadrupole Moments of Nuclei
1.5.1 Quadrupole Moments of Nuclei
1.6 Schmidt Lines
1.7 Semi-Empirical Mass Formula and Its Application to Mass Parabolas
1.8 Mirror Nuclei and Isotopic Spin Formalism
1.8.1 Basic Properties of Nuclei
1.9 Answers to 'Check Your Progress'
1.10 Summary
1.11 Key Terms
1.12 Self Assessment Questions and Exercises
1.13 Further Reading

1.0 INTRODUCTION

The nucleus of an atom, like the atom itself, is a bound quantum system and hence can exist in different quantum states characterized by their energies, angular momenta etc. The lowest energy state is known as the ground state and the nuclei normally exist in this state. The properties of the nuclei which will be discussed in this chapter correspond to their ground states and are usually called their static properties in contrast to the dynamic characteristics of the nuclei which are exhibited in the processes of nuclear reactions, nuclear excitation and nuclear decay. The important static properties of the nuclei include their electric charge, mass, binding energy, size, shape, angular momentum, magnetic dipole moment, electric quadrupole moment, statistics, parity and iso-spin.

The magnetic moment of an atomic nucleus is derived from the spin of protons and neutrons and is known as the nuclear magnetic moment. The quadrupole moment causes some modest modifications in the hyperfine structure, although it is mostly a magnetic dipole moment. Although the relationship between the two values is not obvious or easy to calculate, all nuclei with nonzero spin also have a nonzero magnetic moment.

The nuclear magnetic moment of an element changes from isotope to isotope. The nuclear spin and magnetic moment are always 0 for a nucleus in which the number of protons and neutrons are both even in its ground state (lowest energy state). The nucleus often possesses nonzero spin and magnetic moment when there are odd numbers of protons and neutrons. The nuclear magnetic moment is not the sum of nucleon magnetic moments, and this quality is attributed to the tensorial character of the nuclear force, as in the case of the most basic nucleus, deuterium, which contains both proton and neutron.

NOTES

In this unit, you will learn about the methods for determination of nuclear size and their interpretations, binding energy curve for nuclei and its consequences, nuclear spin, magnetic and quadrupole moments of nuclei, Schmidt lines, semi-empirical mass formula and its applications to mass parabolas and mirror nuclei and isotopic spin formulation.

1.1 OBJECTIVES

After going through this unit, you will be able to:

- Discuss the methods for determination of nuclear size and their interpretations
- Explain about the binding energy curve for nuclei and its consequences
- Define nuclear spin
- Analysis magnetic and quadrupole moments of nuclei
- Describe the Schmidt lines
- Understand the basic concept of semi-empirical mass formula and its applications to mass parabolas
- Learn about the mirror nuclei and isotopic spin formulation

1.2 METHODS FOR DETERMINATION OF NUCLEAR SIZE AND THEIR INTERPRETATIONS

Rutherford's theory of α-particle scattering gives us an idea about the smallness of the nuclear size. Later, Rutherford and his collaborators performed scattering experiments with relatively higher energy α-particles and observed departure from Rutherford scattering formula at large angles; *i.e.*, for small impact parameters *b*. When *b* becomes comparable to the nuclear radius *R*, the α-particle begins to feel the effect of the nuclear force.

Rutherford's formula (σ_R) will be different from unity, we get

$$\cot \frac{\theta_c}{2} = \frac{4\pi\varepsilon_0 M v^2 R}{ZZ' e^2}$$

where $Z' = 2$. For $\theta < \theta_c$, $\sigma/\sigma_R = 1$.

By noting the limiting angle θ_c above which anomalous scattering takes place ($\sigma/\sigma_R \neq 1$), Rutherford estimated the values of the nuclear radius *R* for a few light elements *e.g.*, magnesium. These were of the order of a few times 10^{-15} m.

NOTES

These estimates of Rutherford were not very accurate. In later years more accurate methods for the measurement of the nuclear radius have been developed. It should be noted that when we talk of the nuclear radius, we assume that the nucleus has a spherical shape. This is expected because of the short range character of the nuclear force. However, small departures from the sphericity of certain nuclei have been observed. This is inferred from the existence of electric quadrupole moment of these nuclei which is zero for the spherical nuclei. The departure from sphericity is however small.

In the above discussion, it has been assumed that the nuclear charge is uniformly distributed. Experiments show that this is very nearly so and the nuclear charge density ρ_c is approximately constant. Experimental evidences also show that the distribution of nuclear matter (*i.e.*, protons and neutrons) is nearly uniform, so that the nuclear matter density ρ_m is also approximately constant. Since nuclear mass is almost linearly proportional to the mass number A, this means that

$$\rho_m \sim A/V = \text{constant}$$

i.e., the nuclear volume $V \propto A$. Assuming a spherical shape of the nucleus with a radius R, we then get

$$V = \frac{4}{3}\pi R^3 \propto A$$

or,
$$R \propto A^{1/3}$$

so that
$$R = r_0 A^{1/3} \qquad ...(1.1)$$

where r_0 is a constant, known as the *nuclear radius parameter*.

It should be noted that the nuclear radius, as discussed above, is the radius of nuclear mass distribution. We may also talk about the radius or nuclear charge distribution. Since the nuclear charge parameter (*i.e.*, the atomic number) Z is almost linearly proportional to the mass number A and the nuclear charge density ρ_c is approximately the same throughout the nuclear volume, the distribution of the nuclear charge $+Ze$ should follow the pattern of nuclear mass distribution. Hence the *charge radius* and the *mass radius* of the nucleus may be expected to be very nearly the same. This is due to the strong attractive forces within the nucleus. There are strong evidences to show that this is very nearly the same for both types of nucleons, *viz.*, the protons and the neutrons and hence their distributions within the nuclear volume follow the same pattern.

We now consider the potential energy diagram shown in Fig. 1.1, for a charged particle like a proton or an α-particle, which is acted upon by the electrostatic repulsive force of the nuclear charge $+Ze$ when it is outside the nucleus ($r > R$), while inside the nucleus ($r < R$) a negative potential due to the short range of the specifically nuclear force acts upon it. Here r is the distance from the nuclear centre. We assume arbitrarily that electrostatic force is not effective inside the nucleus, while the nuclear force becomes zero at the nuclear surface $r = R$.

NOTES

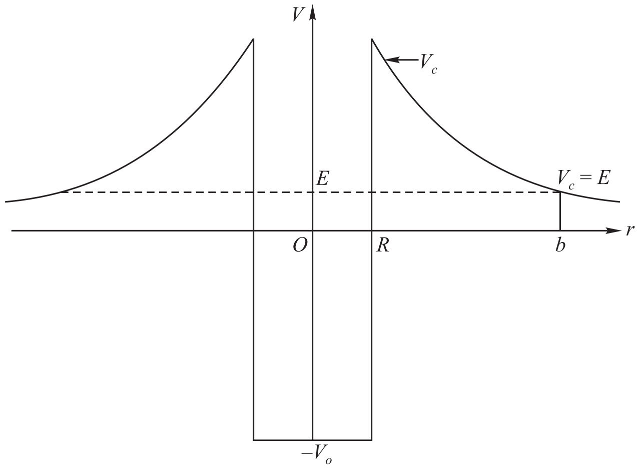

Fig. 1.1 *Potential energy diagram for a nucleus.*

Figure 1.1, shows that the nucleus is surrounded by a Coulomb potential barrier $V_c = ZZ'e^2/\ 4\pi\varepsilon_0\ r$ for an incident particle of charge $Z'e$ for $r > R$. At the nuclear surface the barrier height is given by

$$V_R = \frac{ZZ'e^2}{4\pi\varepsilon_0 R} \qquad ...(1.2)$$

For the uranium nucleus with $Z = 92$ and $R = 8 \times 10^{-15}$ m , $V_R \approx 16.5$ MeV for a proton, while $V_R \approx 33.1$ MeV for an α-particle taking $r_0 = 1.3 \times 10^{-15}$ m.

Classically, a charged particle of energy E less that V_R cannot escape from the nucleus, nor can enter it from outside. However quantum mechanically, because of the uncertainty principle, the position of the particle within the nucleus is not so well-defined, so that there is a finite probability of the particle penetrating through the barrier if E, V_R. If somehow the particle with an initial energy $+E$ outside the nucleus reaches the point $r = b$ where $V_c = E$, then it will be repelled by the electrostatic force of the positive charge of the residual nucleus and will fly away from the latter. We have seen, can account for the α-disintegration of the heavy nuclei.

The radius R, as defined above is usually known as the potential radius, as distinct from the charge of mass radius discussed previously and is slightly larger than the latter.

The charge radius is the most directly measurable one. It can be determined by several methods of which the method based on the scattering of high energy electrons (> 100 MeV) is the most accurate (see below). Besides, there are several other methods. The potential radius must be separately determined, since neither the nature of the specifically nuclear force nor its range is known precisely.

The nuclear radius is usually expressed in units of 10^{-15} m which by international convention is known as the femtometer, abbreviated as *fm* though the unofficial name *fermi* is more often used.

The mean squared radius of nuclear charge distribution can be defined as follows:

$$\langle r^2\rangle = \frac{\int_0^\infty r^2 .4\pi r^2 \rho(r)\,dr}{\int_0^\infty 4\pi r^2 \rho(r)\,dr} \quad ...(1.3)$$

where ρ (*r*) is the nuclear charge density. For a uniformly charged sphere (ρ = constant) of radius *R*, this gives (since ρ = 0 for *r* > *R*)

$$\langle r^2\rangle = \frac{\int_0^R r^4\,dr}{\int_0^R r^2\,dr} = \frac{3}{5}R^2$$

so that

$$R^2 = \frac{5}{3}\langle r^2\rangle \quad ...(1.4)$$

Measurement of the Charge Radius

(*i*) *Electron Scattering Experiment:*

Scattering of high energy electrons by nuclei constitutes the most direct method of measuring the charge radius of the nucleus and the nature of the nuclear charge distribution. This is because there is no specifically nuclear force acting on the electrons. Only the Coulomb attractive force due to the nuclear charge acts on them. If the de Broglie wavelength of the electrons is small compared to the nuclear radius, then the electron scattering experiment can reveal many details of the nuclear charge distribution.

Now according to de Broglie's theory of wave-corpuscular dualism, the wavelength of a relativistic electron of rest mass m_0, having the total energy $E > m_0c^2$ is given by

$$\lambda = \frac{c\,h}{e\left\{V\left(V + 2m_0c^2/e\right)\right\}^{1/2}}$$

where $eV = E_k$ is the kinetic energy of the electron, *e* being its charge. Substituting the values of *c*, *h*, *e* and m_0, we get

$$\lambda = \frac{12.4\times 10^3}{\left\{V\left(V + 1.02\times 10^6\right)\right\}^{1/2}}\ \text{Å}$$

where *V* is in volts. For electrons of kinetic energy E_k = 200 MeV, *V* = 200 × 10^6 volts, which gives

$$\lambda = 6.19 \times 10^{-5}\ \text{Å}$$

and

$$\lambda\!\!\!^{-} = \frac{\lambda}{2\pi} \approx 10^{-15}\ \text{m} = 1\ \text{fm}$$

This is considerably smaller than the radius of most nuclei.

This shows that the use of electrons of a few hundred MeV energy can reveal considerable details regarding nuclear charge distribution.

NOTES

The pioneering experiments on the elastic scattering of electrons by nuclei were carried out by R. Hofstadter and his group at Stanford University in the U.S.A. using the linear accelerator (SLAC), providing electron beam with energy upto 550 MeV. Their experimental arrangement is shown in Fig. 1.2.

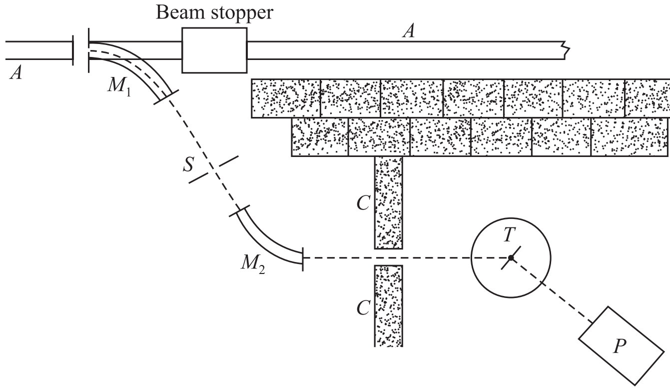

Fig. 1.2 *High energy electron scattering experiment. A-Accelerator; B-Beam stopper; M_1, M_2-Deflecting magnets; S-Collimating slits; T-Scattering chamber; P-Spectrometer; C-Concrete shielding.*

The high energy electron beam from the linear accelerator A is deflected by means of the magnet M_1 and collimated by the slit system S. The deflecting magnet M_2 then directs the beam on to the target inside the scattering chamber T. The elastically scattered beam of electrons is then analysed by the large magnetic spectrometer P.

The quantum mechanical expression for the differential scattering cross-section of a relativistic electron from a spin-less target at the centre of mass angle θ is given by

$$\sigma(\theta) = \sigma_M(\theta)\ \{F(q)\}^2 \qquad ...(1.5)$$

where $\sigma(\theta)$ is the scattering cross section and $\sigma_M(\theta)$ is the Mott cross section of elastic scattering from a point charge $+Ze$ and is given by

$$\sigma_M(\theta) = \left(\frac{Ze^2}{8\pi\varepsilon_0 E}\right)^2 \frac{\cos^2 \theta/2}{\sin^4 \theta/2} \qquad ...(1.6)$$

E is the energy of the electrons in the C.M. system. Eq. (1.6) is valid for low Z elements only. $F(q)$ is called the form factor which gives the ratio by which the scattering cross-section is reduced when the charge $+Ze$ is spread out over finite volume. Because of the destructive interference between the electron waves scattered from different parts of the target nucleus, $F(q) < 1$. Using the Born approximation method of quantum mechanics, it can be shown that

$$F(q) = \frac{1}{Ze}\int \rho(r)\ \exp\ (i.q.r)d\tau \qquad ...(1.7)$$

$$= \frac{4\pi}{Zeq}\int \rho(r)(\sin q\, r)\, r\, dr$$

NOTES

where $$q = k - k' = \frac{1}{\hbar}(p - p') \quad ...(1.8)$$

is a measure of the momentum transfer $p - p'$ in elastic scattering.

$|q|$ depends on the angle of scattering and is given by

$$|q| = \frac{2p}{\hbar}\sin\frac{\theta}{2} \quad ...(1.9)$$

$\rho(r)$ is the charge density within the nucleus and the exponential is a phase factor over the volume. The form factor $F(q)$ is obviously equal to the Fourier transform of the charge density. It can be determined directly by scattering experiment from the ratio $\sigma(\theta)/\sigma_M(\theta)$. Then by using the inverse Fourier transformation, it is possible to determine $\rho(r)$. This is possible if measurements are made at a sufficiently large number of angles θ. When this is not possible, a form of the density distribution, has to be assumed and best fit with the experimental data obtained by suitably adjusting the parameters in the expression. A particularly suitable form for $\rho(r)$ is given by

$$\rho(r) = \frac{\rho_0}{1 + \exp\left\{\left(r - R_{1/2}\right)/a\right\}} \quad ...(1.10)$$

This is known as the *Fermi distribution*. The parameters $R_{1/2}$ and a are adjusted to get the best fit with the experimental data. The above density distribution has the form shown in Fig. 1.3.

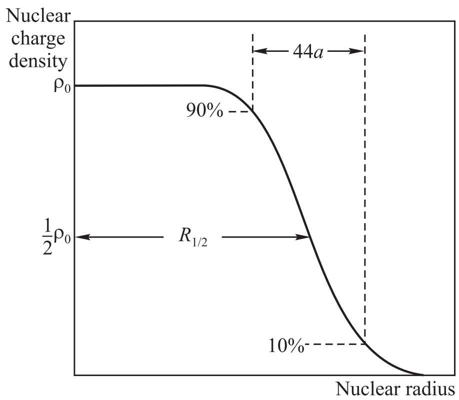

Fig. 1.3 *Fermi distribution for the nuclear charge density.*

Obviously for $r = R_{1/2}$, $\rho = \rho_0/2$ where ρ_0 is the charge density at the centre ($r = 0$). Thus $R_{1/2}$ is the half-value radius. The parameter a determines the skin-thickness of the nucleus, which is the thickness in which $\rho(r)$ falls from 0.9 ρ_0 to 0.1 ρ_0 at the nuclear surface. This comes out to be $t = 4.4\ a$.

If we approximate the above distribution by a uniform charge distribution, then the corresponding equivalent radius can be written as

$$R = r_0 A^{1/3}$$

where $r_0 = 1.32 \times 10^{-15}$ m for $A < 50$ and $r_0 = 1.21 \times 10^{-15}$ m for $A > 50$. This confirms that nuclear matter is distributed almost uniformly within the nuclear volume, if we assume that the mass and charge radii are equal.

The value of a is taken to be the same for all nuclei:

$$a \approx 0.5 \times 10^{-15} \text{ m} = 0.5 \text{ fm}.$$

NOTES

The mass of experimental data so far collected shows that for the spherical nuclei with $A > 15$, the charge distribution has a core of uniform density, surrounded by a skin of thickness 2.3 fm. The radius of half the maximum density $R_{1/2} = 1.07\ A^{1/3}$ fm. For $4 \le A \le 15$, there is no uniform core and the density decreases steadily with increasing r. There is some indication that for all nuclei there is slight diminution in the density near the centre. Further, the charge density in the core region decreases somewhat as Z increases.

As stated before, the distribution of nuclear matter is very similar to that of nuclear charge. In Figs. 1.4 (a) and (b) we compare the nuclear charge and nuclear mass distributions for the three nuclei $^{16}_{8}O$, $^{109}_{47}Ag$ and $^{208}_{82}Pb$. In Table 1.1 are shown the different parameters for nuclear matter distribution.

Table 1.1

Nucleus	$R_{1/2}$ ***(fm)***	***a (fm)***	$R/A^{1/3}$ ***(fm)***
^{16}O	2.61	0.513	1.04
^{109}Ag	5.33	0.523	1.12
^{208}Pb	6.65	0.526	1.12

Assuming a uniform mass distribution, if we write $A = \frac{4\pi}{3} R^3 \rho_m$ then the experimental data gives the radius of uniform mass distribution $R = 1.1A^{1/3}$ fm and $\rho_m = 0.17$ nucleon per fm^3. The nuclear mass density is approximately the same at the centre for all nuclei. It increases slightly with A and tends to the limiting value of 0.17 nucleon/fm^3.

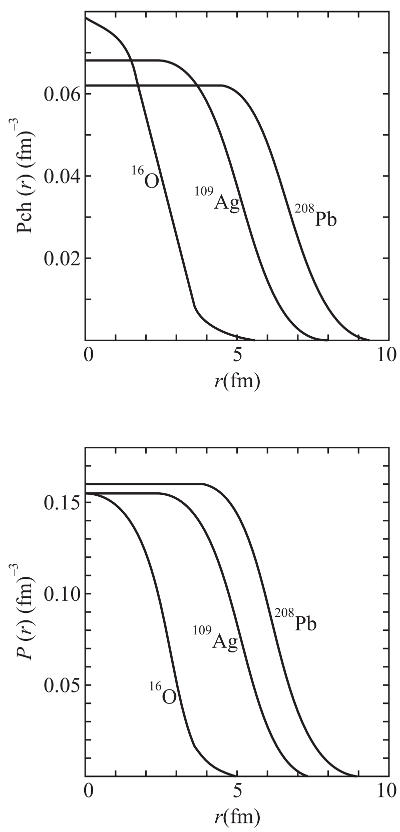

Fig. 1.4 *Experimentally determined (a) nuclear charge and (b) nucleon distributions.*

NOTES

(*ii*) *Muonic x-ray Method:*

There are some alternative methods of determining the mean squared radius of the nuclear charge distribution. One of these is based on the study of the so called mesonic x-rays.

There is a large number of unstable fundamental particles, both charged and neutral, which are observed in nature (usually in the cosmic rays) or can be produced in the laboratory in high energy interactions. One of these is the *muon* (previously called the μ-meson). The muons carry one unit of electronic charge. Both positive and negative muons μ^+ and μ^- are known. They are heavier than the electrons, having rest-mass about 207 m_e where m_e is the electronic mass. They are subjected to the same type of interaction with the nuclei as the electrons, so that only the electrostatic Coulomb force due to the nuclear charge acts on them. Specifically nuclear force (strong interaction) does not act on them.

When a beam of μ^- is passed through matter, some of them are readily captured in electron-like orbits round the nuclei of the capturing atoms forming a muonic atom. The radii of the muonic orbits are however much smaller than the electronic orbits, being smaller by the factor $m_e/m_\mu \sim 1/207$.

We know from Bohr's theory of the spectra of hydrogen-like atom that the radius of the n^{th} electronic orbit is

$$r_e = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_e Ze^2}$$

where Ze is the nuclear charge. So the radius of the muonic orbit should be

$$r_\mu = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_\mu Ze^2} \qquad \text{...(1.11)}$$

Here it is assumed that the nuclear charge e is concentrated at the centre. For a heavy element like gold ($Z = 79$), the radius of muonic K-orbit ($n = 1$) will then be

$$r = \frac{m_e}{m_\mu Z} \times 0.529 \text{ Å} = 3.23 \times 10^{-5} \text{ Å} = 3.23 \times 10^{-15} \text{ m}$$

This is much smaller than the radius of the gold nucleus which is

$$R(\text{Au}) = r_0 A^{1/3} = 1.2 \times 10^{-15} \times (197)^{1/3} = 7 \times 10^{-15} \text{ m}$$

Thus the muonic K-orbit may be expected to lie wholly inside the nucleus in the case of heavy atoms.

When the muon is captured by an atom, it passes from the loosely bound outer orbits into the more tightly bound inner orbits. During the process, electromagnetic radiation is emitted. However the energy of such radiation is much higher than in the case of electronic transitions. The energy of the μ^- in the nth orbit will be on point nucleus assumption

$$E = -\frac{m_\mu Z^2 e^4}{32\pi^2\varepsilon_0^2 n^2\hbar^2} \qquad \text{...(1.12)}$$

Thus in the K-orbit of the gold atom, the orbital energy of μ^- will be

$$E(\text{Au}) = -13.6 \times \frac{m_\mu}{m_e}.Z^2 = -17.6 \text{ MeV}$$

NOTES

This shows that the radiation emitted in the transitions in a muonic atom will lie in the extremely short wavelength x-ray region. From a measurement of these x-ray energies it is possible to estimate the binding energies of the muon in different orbits. However the binding energy in a particular orbit will be greatly reduced if the nuclear charge is spread over a finite region, so that a part of the captured muonic wave function lies within the nucleus. As we have seen above, this is expected for the heavier nuclei. The above mentioned reduction in energy from that expected for a point nuclear charge can be theoretically correlated to the mean squared radius of the nuclear charge distribution. As an example, for Pb atom, the transition $2P_{3/2} \rightarrow 1s_{1/2}$ results in the emission of e.m. radiation of energy 6.02 MeV while that expected on point nucleus hypothesis is 16.4 MeV. The calculations are usually made on the assumption of a specific nuclear charge distribution.

The nuclear radius parameter estimated from muonic x-ray measurements are in reasonable agreement with the electron scattering experiments:

$$r_0 = (1.15 \pm 0.03) \times 10^{-15} \text{ m} = 1.15 \pm 0.03 \text{ fm}$$

(*iii*) *Mirror Nucleus Method:*

The third method of estimating the charge radius of a nucleus is based on the study of the energetics in the β^+ transformation of the mirror nuclei.

Pairs of isobaric nuclei, such as ${}^{11}_{6}C$ and ${}^{11}_{5}B$, ${}^{13}_{7}N$ and ${}^{13}_{6}C$ etc. are known as *mirror nuclei*. The proton number (Z) and the neutron number (N) in them are interchanged and differ by one unit, so that their mass number is $A = 2$, $Z - 1$ where Z is the atomic number of the first member of the pair, the other having the atomic number $(Z - 1)$. The first member of the pair is usually β^+ active and undergoes β^+ transformation into the second.

All nuclear masses can be fairly well represented by a semi-empirical formula, known as the Bethe-Weizsäcker mass formula, which contains a term depending on the Coulomb repulsion between the protons. If the β^+ transformation energy (Q_{β^+}) is calculated using this formula, then Q_{β^+} is found to vary linearly with $A^{2/3}$, the constant of proportionality depending on the value r_0, the nuclear radius parameter.

r_0 estimated from these studies is found to agree fairly well with those estimated by the other methods discussed earlier.

$$r_0 = (1.28 \pm 0.05) \times 10^{-15} \text{ m} = 1.28 \pm 0.05 \text{ fm}$$

The different methods of measurement of the charge radius give a mean value of the radius parameter $r_0 = (1.19 + 0.1\, A^{-1/2})$ fm. As can be seen, this is slightly dependent on A.

Measurement of Potential Radius

The specifically nuclear force is a strong short-range force. The potential from

NOTES

which this force is derived is thus of short range and has a steep slope at the edge of the nucleus. It owes its origin to the strong short range internucleon interaction. There are evidences to indicate that this is independent of the nature (*i.e.*, charge state) of the nucleons, so that the $p-p$ and $n-n$ forces are equal (charge symmetry). In addition, the $p-n$ force is also the same in the same quantum state ($'S$): Obviously for a complex nucleus, the specifically nuclear interaction will extend upto a distance of the same order of magnitude as the range of the internucleon interaction beyond the radius R_0 of nuclear charge distribution. This is the radius shown in the potential energy diagram (Fig. 1.1) and is known as the *potential radius*, which is thus slightly larger than R_0. We discuss below two different methods of estimating the potential radius.

(*i*) *Life Time of Alpha Emitters:*

Historically the earliest method of estimating the potential radius was based on the study of alpha-disintegration of heavy nuclei like ^{238}U, ^{226}Ra etc. Alpha-disintegration of heavy nuclei takes place due to the penetration of the Coulomb potential barrier surrounding the nucleus. According to this, the barrier penetration probability (transmission co-efficient) is given by

$$T = \exp(-G) \quad ...(1.13)$$

where

$$G = \frac{2}{\hbar}\left(\frac{MZe^2b}{\pi\varepsilon_0}\right)^{1/2}\left\{\cos^{-1}\sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b}-\frac{R^2}{b^2}}\right\} \quad ...(1.14)$$

where R is the nuclear radius (potential radius) and b is the distance from the centre to the point where the energy E of the α-particle is equal to the Coulomb potential energy $V_c = 2\,Ze^2/4\pi\varepsilon_0 r$. Here Z is the atomic number of the residual nucleus. M and $2e$ are the mass and the charge of the α-particle; r is measured from the centre of the nucleus.

If n be frequency of collision of the α-particle against the nuclear wall inside the nucleus, then the probability of penetration through the barrier per second is $p = nT$. The reciprocal of this is the mean life of α-decay which can be measured:

$$\tau_m = \frac{1}{p} = \frac{1}{nT} \quad ...(1.15)$$

Thus by measuring the mean life it is possible to estimate the potential radius R. Writing $R = r_0 \,.\, A^{1/3}$ as before, the potential radius parameter is found to be $r_0 = 1.48 \times 10^{-15}$ m.

It should be noted that though the above theoretical formula does not reproduce the α-decay life times accurately and may deviate by several orders of magnitude from the experimental value, it gives a much more precise estimate of the nuclear radius R, even from a rough knowledge of τ_m.

r_0 estimated by this method is somewhat higher than that for the charge or mass radius parameter. A correction due to the finite radius of the α-particle ($R_\alpha - 1.2 \times 10^{-15}$ m) gives the radius of the residual nucleus R_A such that $R = R_A + R_\alpha$ where R_A can be expressed'by the formula

$$R_A = r_{0A} A^{1/3}$$

The new parameter $r_{0A} = 1.4 \times 10^{-15}$ m ...(1.16)

(*ii*) *Neutron Scattering Experiments:*

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In these experiments, mono-energetic beams of fast neutrons are allowed to be scattered by nuclei. Since neutrons interact mainly by the strong specifically nuclear interaction with the nucleus, this method actually detects the edge of the nuclear potential well. It can be shown that the total cross-section for fast neutrons is given by

$$\sigma_T = 2\pi (R + \lambda\!\!\!^{-})^2 \approx 2\pi R^2 \quad ...(1.17)$$

where the de Broglie wavelength $\lambda << R$, which happens at high energies, $\lambda\!\!\!^{-}$ being equal to $\lambda/2\pi$. Also at such high energies, the neutron absorption cross section is given by

$$\sigma_a = \pi R^2$$

assuming a perfectly black nucleus which absorbs all the neutrons falling on it.

The measurements of the above cross-sections give a radius parameter $r_0 = 1.25 \times 10^{-15}$ m.

Neutron measurements are usually difficult. So measurements using charged particles which interact strongly with the nuclei at close range, such as α-particles or protons upto a few hundred MeV have also been made. In the α-particle experiments, the critical angle of scattering at which deviations are observed from the Rutherford scattering is measured. θ_c can be correlated with the critical distance at which the effect of the specifically nuclear force begins to be felt .

In proton elastic scattering experiments (5 to 200 MeV), diffraction patterns are observed due to the extension of the potential beyond the nuclear edge. A specific form of the nuclear potential is assumed to fit the experimental data. The following potential form due to Woods and Saxon (optical potential) is usually employed to analyse the data (see Ch. XI):

$$V(r) = \frac{V_0}{1 + \exp\left\{\left(r - R_{1/2}\right)/a\right\}} \quad ...(1.18)$$

This has a radial dependence similar to the Fermi charge distribution discussed in this book. $R_{1/2}$ and a have the same meanings as before.

A value $r_0 = 1.33 \times 10^{-15}$ m is derived from the experimental data.

The potential radius is about 0.7 fm greater than the charge radius which may be taken to be the measure of the range of nuclear force.

We can summarize the results of the different types of measurements as below:

(*a*) Mass distribution: $r_{0m} = 1.1 \times 10^{-15}$ m.

(*b*) Equivalent square well for charge distribution: $r_{0c} = (1.2 \text{ to } 1.3) \times 10^{-15}$ m.

(*c*) Optical potential: $r_{0v} = 1.25 \times 10^{-15}$ m.

Check Your Progress

1. What important work did Rutherford do?
2. State the electron scattering experiment.
3. What do you understand by the μ–meson?
4. Define mirror nucleus method.
5. What do you mean by life time of alpha emitters?

NOTES

1.3 BINDING ENERGY CURVE FOR NUCLEI AND ITS CONSEQUENCES

Accurate determination of the atomic masses shows that these are very close to whole numbers, which are actually the mass numbers of the atoms, when the masses are expressed in the units of atomic masses in the ^{12}C scale. The same is also true if the atomic masses are expressed in ^{16}O scale.

Considering the ^{12}C scale, the atomic mass of ^{12}C is exactly 12 u. The masses of all other atoms, though close to the corresponding mass numbers (integral), differ slightly from the latter.

The masses of a few atoms listed in Table 1.2 below will bear this out.

Table 1.2

Atom	*Atomic Mass* (*u*)	*Mass Defect* (*u*)	*Packing Fraction* (*u*)
^{1}n	1.008665	+ 0.008665	–
^{1}H	1.007825	+ 0.007825	–
^{2}H	2.014102	+ 0.014102	+ 0.007051
^{4}He	4.002603	+ 0.002603	+ 0.00006507
^{12}C	12	0	0
^{16}O	15.994915	– 0.005085	– 0.0003178
^{31}P	30.973764	– 0.026236	– 0.0008463
^{59}Co	58.933189	– 0.066811	– 0.0011324
^{75}As	74.921597	– 0.078403	– 0.0010454
^{127}I	126.90447	– 0.09553	– 0.0007522
^{197}Au	196.96654	– 0.03346	– 0.0001698
^{226}Ra	226.02543	+ 0.02543	+ 0.0001125
^{238}U	238.05082	+ 0.05082	+ 0.0002135

The table shows that for very light atoms with $A < 20$ and for very heavy atoms with $A > 180$, the atomic masses are slightly greater than the corresponding mass numbers. In between the above values of A, the atomic masses are slightly less than the corresponding mass numbers.

The departure of the measured atomic mass $M(A, Z)$ from the mass numbers (A) is quite significant. The difference between M and A is known as the mass defect ΔM :

$$\Delta M = M(A, Z) - A \qquad ...(1.19)$$

NOTES

For example since the atomic mass of ^{4}He (4.002603 u) is slightly greater than the mass number 4, its mass defect is + 0.002603 u. On the other hand ^{75}As has the atomic mass 74.9215967 u, which is slightly less than the mass number 75. Its mass defect is – 0.078403 u. Thus the mass defect can be both positive and negative. For very light and very heavy atoms, the mass defect is positive, while in the intermediate region it is negative (Refer Table 1.2).

The mass defect of an atom divided by its mass number is known as the *packing fraction* (f), a term introduced by F.W. Aston. Thus

$$f = \frac{\Delta M}{M}$$

$$= \frac{M(A,Z)}{A} - 1 \qquad ...(1.20)$$

In the last column of Table 1.2, the packing fractions of the different atoms are listed. f has the same sign as ΔM and is positive for very light and very heavy atoms. It is negative for the atoms in the intermediate region.

From Eq. (1.20), we have

$$M(A, Z) = A\,(1 + f)$$

It is found that the packing fraction f varies in a systematic manner with the mass number A. The nature of this variation is shown graphically in Fig. 1.5.

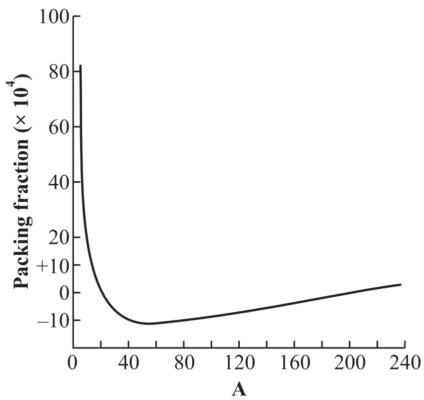

Fig. 1.5 *Packing fraction curve.*

From the figure it is seen that for very light nuclei the packing fraction is positive and decreases rapidly with increasing A. It becomes negative for A greater than about 20, attains a minimum (negative) at $A \sim 60$. It then rises slowly for higher A and becomes positive again for A greater than about 180.

This systematic variation of f with A can be understood from nuclear binding energy considerations.

If the binding energy E_B of a nucleus ^A_ZX defined by equation discuss in this book is divided by the mass number A, we get the binding energy per nucleon in the nucleus, which is known as the *binding fraction* (f_B) and is given by

$$f_B = \frac{E_B}{A} = \frac{ZM_H + NM_n - M(A,Z)}{A} \qquad ...(1.21)$$

NOTES

Here we have assumed that the masses are expressed in energy unit so that c^2 on the r.h.s. of Eq. (1.21) has been omitted.

We can estimate the values of f_B for a few typical cases, using the mass values given in Table 1.2.

For deuteron (^{2}H), since $Z = 1$, $N = 1$,

$$E_B\,(^2\text{H}) = M_H + M_n - M_d$$
$$= (1.007825 + 1.008665 - 2.014102) \times 931.5$$
$$= 2.224 \text{ MeV}$$

$$\therefore \quad f_B\,(^2\text{H}) = \frac{2.224}{2} = 1.112 \text{ MeV per nucleon}$$

For the α-particle (^{4}He), since $Z = 2$, $N = 2$,

$$E_B\,(^4\text{He}) = (2 \times 1.007825 + 2 \times 1.008665 - 4.002603) \times 931.5$$
$$= 28.3 \text{ MeV}$$

$$\therefore \quad f_B\,(^4\text{He}) = \frac{28.3}{4} = 7.075 \text{ MeV per nucleon}$$

For (^{16}O) nucleus, since $N = 8$, $Z = 8$,

$$E_B\,(^{16}\text{O}) = (8 \times 1.007825 + 8 \times 1.008665 - 15.994915) \times 931.5$$
$$= 127.62 \text{ MeV}$$

$$\therefore \quad f_B\,(^{16}\text{O}) = \frac{127.62}{16} = 7.98 \text{ MeV/nucleon}$$

The binding fractions of the different nuclei represent the relative strengths of their binding. Thus ^{2}H is very weakly bound, compared to ^{4}He or ^{16}O. The nature of variation of f_B for the different nuclei with A is shown graphically in Fig. 1.6.

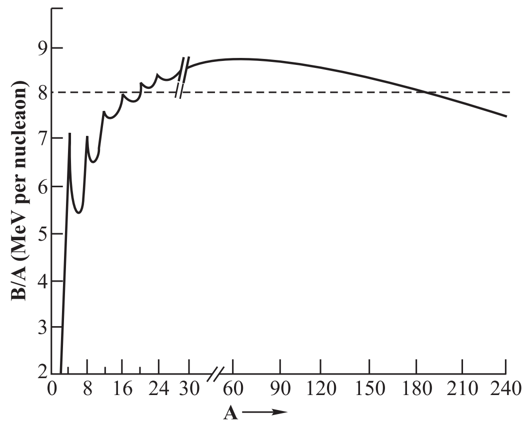

Fig. 1.6 *Binding fraction curve.*

The following points about the variation of f_B against A are noteworthy: (*a*) f_B for the very light nuclei is very small and rises rapidly with A attaining a value of ~ 8 MeV/nucleon for $A \sim 20$. It then rises slowly with A and attains a maximum of 8.7 MeV per nucleon at $A \sim 56$. For higher A, it decreases slowly. (*b*) For $20 < A < 180$, the variation of f_B is very slight, so that it may be taken to be approximately constant in this region having a mean value of ~ 8.5 MeV per nucleon. (*c*) For very heavy nuclei ($A > 180$), f_B decreases monotonically with the increase of A. For the heaviest nuclei, f_B is about

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7.5 MeV/nucleon. (*d*) For very light nuclei, there are rapid fluctuations in the values of f_B. In particular, peaks are observed in the f_B *vs.* A graph for the even-even nuclei ^{4}He, ^{8}Be, ^{12}C, ^{16}O etc., for which $A = 4n$ where n is an integer. Similar, but less prominent peaks are observed at the values of Z or N equal to 20, 28, 50, 82 and 126. These are known as *magic numbers*.

The appearance of the peaks shows greater stability of the corresponding nuclei relative to the nuclei in their immediate neighbourhood.

The nature of the binding fraction curve is complementary to the nature of the packing fraction curve (Fig. 2.1). The reason for this can be understood as follows. If we write $M_H = 1 + f_H$ and $M_n = 1 + f_n$ where $f_H = 0.007825$ u and $f_n = 0.008665$ u are constant, then we have

$$\begin{aligned} E_B &= Z(1 + f_H) + N(1 + f_n) - M(A, Z) \\ &= (Z + N) + Zf_H + Nf_n - A(1 + f) \\ &= A + Zf_H + Nf_n - A - \Delta M \end{aligned}$$

where $\Delta_M = A_f$. Hence we get

$$E_B = Zf_H + Nf_n - \Delta M \qquad \text{...(1.22)}$$

$$\therefore \qquad f_B = \frac{E_B}{A} = \frac{Zf_H + Nf_n}{A} - \frac{\Delta M}{A}$$

$$= \frac{Zf_H + Nf_n}{A} - f \qquad \text{...(1.23)}$$

The first term on the r.h.s. of Eq. (1.23) is almost a constant specially for lower A when $Z \approx N \approx A/2$.

Thus f_B increases or decreases as f decreases or increases respectively. Hence the graphs of variation of f and f_B with A have complementary appearances. Corresponding to the minimum in the graph of f *vs.* A, there is a maximum in the graph of f_B *vs.* A. Also the region of negative slope for low A in the first case, corresponds to the region of positive slope in the second case. For higher A on the other hand, the region of positive slope in the first case corresponds to the region of negative slope in the second.

With the help of the binding fraction curve it is possible to explain in a qualitative manner the reasons for the α-disintegration of heavy nuclei as also of the energy release in nuclear fission and fusion processes. These will be discussed at appropriate places.

1.4 NUCLEAR SPIN

As stated before, a complex nucleus is made up of protons and neutrons, collectively known as nucleons. The protons and neutrons have intrinsic spin angular momentum 1/2(in unit of $\hbar$) each, just like the electrons. In addition, the nucleons also possess quantized orbital angular momenta about the centre of mass of the nucleus, like the electrons in the atom. The resultant angular momentum **I** of the nucleus is thus the vector sum of the orbital angular momentum **L** and spin angular momentum **S** of the nucleus:

$$\mathbf{I} = \mathbf{L} + \mathbf{S} \qquad \text{...(1.24)}$$

NOTES

Quantum mechanical considerations show that the total orbital and spin angular momenta of the nucleus are given by

$$p_I^2 = I\,(I + 1)\,\hbar^2$$
$$p_L^2 = L\,(L + 1)\,\hbar^2$$
$$p_S^2 = S\,(S + 1)\,\hbar^2$$

During measurement, it is the largest component of the angular momentum along the direction of the applied electric or magnetic field which is determined. For the three cases mentioned above, these have the magnitudes I, L and S respectively in units of $\hbar$.

The resultant spin angular momentum of the nucleus is obtained by the vector addition of the spins of the individual nucleons : $S = \sum s_i$. Similarly, the resultant orbital angular momentum is given by $L = \sum l_i$. Since $s_i = 1/2$, S can be either integral or half-integral, depending on whether the number of nucleons A in the nucleus is even or odd. On the other hand, since l_i is integral (0, 1, 2, etc.), L is integral or zero. Thus the total angular momentum I of the nucleus can be either integral (for A even) or half odd integral (for A odd). This is in agreement with observations.

The total nuclear angular momentum I is usually referred to as the nuclear spin. Measurements of the ground state spin of nuclei show that for even Z even N nuclei, the nuclear spin is invariably zero ($I = 0$). This shows that there is a tendency of the nucleons inside the nucleus to form pairs with equal and oppositely aligned angular momenta, which cancel out in pairs for like nucleons.

Another important point to note is that the measured values of the ground state spins of the nuclei are small integers or half odd integers, the highest measured value being 9/2 which is small compared to the sum of the absolute values of l_i and s_i of all the individual nucleons contained in the nucleus. This is in conformity with what was stated above regarding pair formation within the nuclei. Majority of the nucleons of either type seems to form a core in which even numbers of protons and neutrons are grouped in pairs of zero spin and orbital angular momenta so that the core itself has zero total angular momentum. The few remaining nucleons outside the core determine the nuclear spin which is thus a small number, integral or half odd integral.

Methods of measurement of the ground state spins of nuclei will be discussed in this book. Spins of excited states of nuclei are deduced from nuclear disintegration and nuclear reaction data.

Pauli's Spin Formalism

It may be mentioned here that the spin of a spin 1/2 particle like the nucleons moving non-relativistically is treated in terms of Pauli's theory. Pauli introduced the spin operator σ related to the spin vector s through the relation $s = \left(\frac{\hbar}{2}\right)\sigma$; σ has the three components σ_x, σ_y and σ_z which are 2×2 matrices

(Pauli matrices) as given below

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ...(1.25)$$

NOTES

Then $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$ which is a 2 × 2 unit matrix (1). The two states of the particle (*spin up* and *spin down*) are the two component Pauli spinors

$$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\uparrow, \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\downarrow \quad ...(1.26)$$

Operation of α and β by the Pauli matrices gives the following results, as can be easily verified by direct matrix multiplication:

$$\left.\begin{array}{ll} \sigma_x\alpha = \beta & \sigma_x\beta = \alpha \\ \sigma_y\alpha = i\beta & \sigma_y\beta = -i\alpha \\ \sigma_z\alpha = \alpha & \sigma_z\beta = -\beta \\ \text{We also have} & \\ \sigma^2\alpha = 3\alpha & \sigma^2\beta = 3\beta \end{array}\right\} \quad ...(1.27)$$

which gives

$$\left.\begin{array}{l} s^2\alpha = \dfrac{3\hbar^2}{4}\alpha = \dfrac{1}{2}\left(\dfrac{1}{2}+1\right)\hbar^2\alpha = s(s+1)\hbar^2\alpha \\ s^2\beta = \dfrac{3\hbar^2}{4}\beta = \dfrac{1}{2}\left(\dfrac{1}{2}+1\right)\hbar^2\beta = s(s+1)\hbar^2\beta \end{array}\right\} \quad ...(1.28)$$

Also $$s_z\alpha = \frac{\hbar}{2}\alpha \qquad s_z\beta = -\frac{\hbar}{2}\beta \quad ...(1.29)$$

Thus α and β are the simultaneous eigen-vectors of s^2 and s_z belonging to the eigen-values $3\hbar^2/4$ and $\pm\hbar/2$ respectively, the plus sign corresponding to the spin up state and the minus sign corresponds to the spin down state.

The components of σ anti-commute, which means

$$\sigma_x\sigma_y + \sigma_y\sigma_x = 0,\ \sigma_y\sigma_z + \sigma_z\sigma_y = 0, \sigma_z\sigma_x + \sigma_x\sigma_z = 0 \quad ...(1.30)$$

We further have

$$\sigma_x\sigma_y - \sigma_y\sigma_x = 2i\sigma_z,\ \sigma_y\sigma_z - \sigma_z\sigma_y = 2i\sigma_x, \sigma_z\sigma_x - \sigma_z\sigma_z = 2i\sigma_y \quad ...(1.31)$$

These give

$$\sigma_x\sigma_y = i\sigma_z,\quad \sigma_y\sigma_z = i\sigma_x, \sigma_z\sigma_x = i\sigma_y \quad ...(1.32)$$

1.5 MAGNETIC AND QUADRUPOLE MOMENTS OF NUCLEI

Like the electron, the proton and the neutron possess intrinsic magnetic dipole moments. The measured values of the magnetic moments of the proton and the neutron are

$$\mu_p = 2.7927\ \mu_N$$

$$\mu_n = -1.9131\ \mu_N$$

where $$\mu_N = e\hbar/2M_p \quad ...(1.33)$$

NOTES

is called the *nuclear magneton*. e and M are the charge and mass of the proton. μ_N is analogous to the Bohr magneton $\mu_B = e\hbar/2m_e$ which is the unit of atomic magnetic moment. μ_N is much smaller than μ_B, being only 1/1836 part of the latter. Since $\mu_B = 9.2849 \times 10^{-24}$ J/T, we get

$$\mu_N = 5.0571 \times 10^{-27} \text{ J/T}$$

The above numerical values show that the proton and neutron magnetic moments are of the order of 10^{-3} times the electronic magnetic moment, which is equal to the Bohr magneton ($\mu_e = \mu_B$). Since the nuclei are made up of protons and neutrons, the magnetic moments of the nuclei are also much smaller than the atomic magnetic moments, the latter being of the order of electronic magnetic moments.

Except for a minor correction, the electronic magnetic moment is correctly predicted by the relativistic quantum mechanical theory of the electron propounded by P.A.M. Dirac. If the motion of the proton is described by the same theory, then the proton should have a magnetic moment $\mu_p = \mu_N$. However this is not so and μ_p is greater than μ_N. Further the neutron being an uncharged particle, is not normally expected to have a magnetic moment. Again this is not true and μ_n has a magnitude greater than μ_N. These anamalous values of the magnetic moments of the proton and the neutron can be understood, at least qualitatively, on the basis of the meson theory.

It should be noted that the magnetic moments of the proton and the neutron are intimately related to their intrinsic spin angular momenta, which are given by

$$p_p = s_p\hbar, P_n = s_n\hbar$$

with $s_p = s_n = 1/2$. The ratio of the magnetic moment μ_e to the spin angular momentum p_e of the electron is given by

$$\frac{\mu_e}{p_e} = g_e \frac{e}{2m_e} \quad ...(1.34)$$

where $p_e = s_e\hbar = \hbar/2$, g_e being the Lande factor. It has the value $g_e = -2$. The quantity of the r.h.s. of the above equation is the gyromagnetic ratio for the spin motion of tile electron. The factor $g_e = -2$ was at first introduced by S. Goudsmit and G.E. Uhlenbeck on ad hoc basis but later found justification from the Dirac electron theory.

In the case of the proton and the neutron, we can write, in analogy with Eq. (1.34)

$$\frac{\mu_p}{p_p} = g_e \frac{e}{2M_p} \quad ...(1.35)$$

$$\frac{\mu_n}{p_n} = g_n \frac{e}{2M_p} \quad ...(1.36)$$

Substituting the values of p_p and p_n, we get

$$\mu_p = g_p \frac{e}{2M_p} .s_p \hbar = \frac{g_p}{2} \mu_N \quad ...(1.37a)$$

$$\mu_n = g_n \frac{e}{2M_n} . s_n \hbar = \frac{g_n}{2} \mu_N \qquad ...(1.37b)$$

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Comparison with Eqs. (1.33) gives

$$g_p = 2 \times 2.7927, g_n = -2 \times 1.9131 \qquad ...(1.38)$$

Eqs. (1.37) can be written in vector forms (in nuclear magnetons) as

$$\overrightarrow{\mu_p} = \frac{1}{2} g_p \sigma_p \qquad ...(1.39a)$$

$$\overrightarrow{\mu_n} = \frac{1}{2} g_n \sigma_n \qquad ...(1.39b)$$

σ_p and σ_n are the Pauli spin operators.

The quantity g_e appearing in Eq. (1.34) is negative because of the negative sign of the electronic charge. Classically, the rotation of the electron constitutes a current opposite to the direction of its rotation. This current loop gives rise to a magnetic moment perpendicular to the plane of the loop directed opposite to the angular momentum associated with the rotation, *i.e.*, μ_e is opposite to p_e so that g_e is negative.

For the proton, because of its positive charge, the directions of μ_p is the same as that of p_p and hence g_p is positive.

The negative sign of g_n then obviously indicates that μ_n is directed opposite to p_n.

For a complex nucleus, the intrinsic magnetic moments of all the protons are to be vectorially added to give the resultant $\sum \overrightarrow{\mu_{pi}}$. Similarly the intrinsic magnetic moments of all the neutrons are to be vectorially added to give the vector $\sum \overrightarrow{\mu_{ni}}$. In addition, the orbital rotations of the protons will also contribute to the net magnetic moment of the nucleus equal to $\sum \left(\overrightarrow{\mu_{lp}}\right)_i$. This last can be defined in the same way as in the case of the orbital motion of the electron. If p_L be the resultant orbital angular momentum due to the orbital motion of the protons, then we can write

$$\frac{\mu_L}{p_L} = g_L \frac{e}{2M_p}$$

Writing $p_L = L\hbar$ we then get

$$\mu_L = g_L \cdot \frac{e\hbar}{2M_p} L = g_L L \mu_N \qquad ...(1.40)$$

L is the orbital angular momentum quantum number. For orbital motion of the proton $g_L = 1$ as in the case of the electron, so that

$$\mu_L = L\mu_N \qquad ...(1.41)$$

L can have only integral values or can be zero.

No contribution to the magnetic moment of the nucleus comes from the orbital motion of the neutrons ($g_L = 0$ for neutrons).

Hence the resultant magnetic moment of the nucleus is obtained by the vector addition of the three vector quantities $\sum \overrightarrow{\mu_{pi}}$, $\sum \overrightarrow{\mu_{ni}}$, and $\sum \left(\overrightarrow{\mu_{lp}}\right)_i$.

NOTES

The protons and the neutrons tend to form pairs with oppositely aligned spins, giving a resultant spin 0. Obviously such pairs will also have zero magnetic moments. Hence the net magnetic moment of the nucleus will be determined by the nucleons outside the even Z—even N core for which the net magnetic moment is zero. As in the case of the nuclear spin, this makes the value of the magnetic moment of the nucleus comparable to the proton or neutron magnetic moments.

1.5.1 Quadrupole Moments of Nuclei

Nuclear electric quadrupole moments can be estimated from observations on the departures from the interval rule, according to which the energy difference between two states F and $F - 1$ for given J (electronic angular momentum quantum number) and I is linearly proportional to F where $F = J + I$. The departures can occur due to two reasons : (*a*) Magnetic perturbations of the nearby levels; (*b*) Effect of the nuclear electric quadrupole moment.

In the second case, the interaction of the nuclear electric quadrupole moment with the electric field of the electrons gives rise to the appearance of additional hyperfine structure lines, which do not obey the interval rule due to magnetic interaction between I and J. The existence of the electric quadrupole moment of the deuteron was discovered by this method. It has the value $Q_d = 2.82 \times 10^{-31}$ m^2.

The electric quadrupole moment of the nuclei can also be estimated from the hyperfine structure of the microwave spectra of the molecules containing the nuclei in several cases.

The electrostatic interaction energy between the nucleus of an atom in the molecule and the remaining charges (electronic and nuclear) depends on the *electric quadrupole coupling coefficient* given by

$$q = eQ\left(\frac{\partial^2 V^e}{\partial z^2}\right)_0 \qquad ...(1.42)$$

where V^e is the potential due to all charges external to the nucleus under consideration.

The second derivative is determined at the position of the nucleus along the symmetry axis of the molecule.

We can obtain an estimate of the interaction energy of a nuclear electric quadrupole in the electric fieid due to all the charges external to the nucleus from the following classical considerations. Assuming the field E to be cylindrically symmetric with the symmetry axis along z and taking into account the variation of the field over the nuclear volume we can write

$$E_z = (E_z)_0 + \left(\frac{\partial E_Z}{\partial z}\right)_0 = Kz \qquad ...(1.43)$$

where we have assumed the field at the nucleus to be zero and have put

$$K = \left(\frac{\partial E_Z}{\partial z}\right)_0 \qquad ...(1.44)$$

NOTES

Since the source producing the field E is away from the nucleus, $\nabla \cdot E = 0$. This is possible if we write

$$E_x = -K\,x/2 \text{ and } E_y = -K\,y/2 \qquad ...(1.45)$$

for a cylindrically symmetric field.

It is shown that the interaction energy of an electric quadrupole in an external electric field is given by

$$U_2 = -1/6 \sum_i \sum_j Q_{ij} \left(\frac{\partial E_j}{\partial x_i}\right)_0 \qquad ...(1.46)$$

where i, j can take up the values 1, 2, 3 each corresponding to the three rectangular coordinates $x = x_1$, $y = x_2$, $z = x_3$. The quadrupole moment tensor Q_{ij} is given by

$$Q_{ij} = \int \rho(r)(3x_i x_j - r^2\delta_{ij})d\tau \qquad ...(1.47)$$

where $\rho(r)$ is the density of the charge distribution. Since Q_{ij} is a symmetric tensor, only the diagonal components $Q_{xx} = Q_{11}$, $Q_{yy} = Q_{22}$ and $Q_{zz} = Q_{33}$ are non vanishing. Also the sum of the diagonal elements is zero:

$$Q_{11} + Q_{22} + Q_{33} = 0 \qquad ...(1.48)$$

So, for an axially symmetric charge distribution (as for an ellipsoid of revolution)

$$Q_{11} = Q_{22} = -Q_{33} = -Q \text{ (say)} \qquad ...(1.49)$$

where $Q = Q_{33}$ is called the quadrupole moment of the charge distribution.

We then have ($\because Q_{ij} = 0$ for $i \neq j$)

$$U_2 = -\frac{1}{6}\left[Q_{xx}\left(\frac{\partial E_x}{\partial x}\right)_0 + Q_{yy}\left(\frac{\partial E_y}{\partial y}\right)_0 + Q_{zz}\left(\frac{\partial E_z}{\partial z}\right)_0\right]$$

$$= -\frac{1}{6}\left(\frac{Q}{2}.\frac{K}{2} + \frac{Q}{2}.\frac{K}{2} + Q.K\right)$$

$$= -\frac{1}{4}Q\left(\frac{\partial E_z}{\partial z}\right)_0 = -\frac{Q}{4}\left(\frac{\partial^2 V^e}{\partial z^2}\right)_0 \qquad ...(1.50)$$

It should be noted that since Q is measured in the unit of an area (m^2) in nuclear physics, the above expression should be multiplied by the unit of charge e.

The quadrupole coupling coefficient q can be determined from observations on the hyperfine splitting in the microwave spectra. However, for the determination of the nuclear electric quadrupole moment Q from the values of q, one must know the value of $(\partial^2 V^e/\partial z^2)$. Various methods have been developed for estimating this quantity. However, there are uncertainties in such estimates which introduce considerable error in the determination of Q.

Linear triatomic molecules, such as ClCN, BrCN, OCS, etc., and symmetric top molecules like CH_3Cl and CH_3Br have been studied by this method.

NOTES

1.6 SCHMIDT LINES

The plot of j μ with *J* is shown in the Figure 1.7, the lines are known as Schmidt lines. The agreement between the experimental and theoretical values may be due to the error in the measurement of magnetic moment or due to the assumption that the nucleons move in a spherical symmetric potential which is not true.

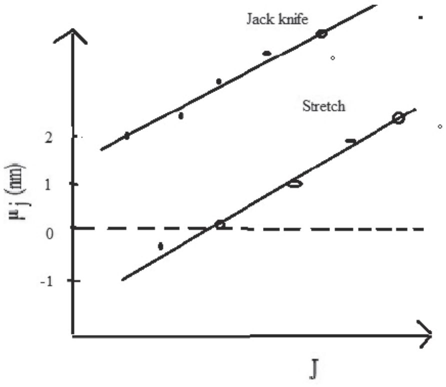


Fig.1.*7 Schmidt Lines*

1.7 SEMI-EMPIRICAL MASS FORMULA AND ITS APPLICATION TO MASS PARABOLAS

Typically, the *binding energy* is expressed as B/A or binding energy per nucleon. This demonstrates that the binding energy is proportional to A in general, because B/A is largely constant. However, there are reversals to this pattern. The *semi-empirical mass formula* captures the dependence of B/A on A (and Z). This formula is based on fundamental principles (a model for the nuclear force) and experimental evidence to determine the precise parameters that define it. In this model, dubbed the *liquid-drop model*, all nucleons are equally distributed within a nucleus and are held together by the nuclear force, whereas protons are repelled by the Coulomb contact. The nuclear force's (short range) and Coulomb interaction properties explain a portion of the *semi-empirical mass formula*. Nonetheless, additional (smaller) modifications have been made to account for changes in the binding energy that arise as a result of its quantum-mechanical origin (and that give rise to the nuclear shell model).

There is a formula called the Semi-Empirical Mass Formula (SEMF). It says:

$$M\,(Z, A) = Z_m(^1H) + Nm_n - B\,(Z, A)/c^2$$

where *B(A, Z)* denotes the binding energy, which may be calculated using the following formula:

NOTES

$$B(A,Z) = a_v A \; - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + \delta a_p A^{-3/4}$$

volume   surface   Coulomb   symmetry   pairing

The following are the terms covered by the SEMF, in order of importance:

1. Volume Term:

The first term is the volume term $a_v A$, which represents the fact that the binding energy is largely related to A. Why is this the case?

If you think about it, nucleon interactions are measured by the binding energy of the nucleons. Weak nuclear forces and dense nuclear packing limit the nucleon's ability to interact with more than a few nearby neighbours. This shows that no matter how many nucleons there are, each one contributes the same amount. To put it another way, the force is not $A(A - 1)/2 \sim A^2$ but rather A (the total number of nucleons with which one nucleon can interact). According to experimental results, the $a_v = 15.5$ MeV proportionality constant is the fitting parameter.

When it comes to nuclear (strong) interactions, however, this number is smaller than what the nucleons can bind to one other. When one nucleon is bound to another, its binding energy is around 50 *MeV*. No, a nucleon's binding energy isn't equal to the sum of its interactions with other nucleons and its own motion. Although there are no nucleons in an atom with zero kinetic energy, Pauli's exclusion rule dictates that they will fill all of the kinetic energy levels in an atom. This model provides an accurate estimate of a_v, which includes nuclear binding energy and the kinetic energy generated by filling shells.

2. Surface Term:

The surface term, $-a_s A^{2/3}$, is a correction to the volume term, as it is also based on the strong force. As previously explained by the volume term, a constant number of nucleons interact with each nucleon. While this holds true for nucleons located deep within the nucleus, nucleons located on the nucleus's surface have fewer nearest neighbours. This word is synonymous with surface forces, which occur in droplets of liquids and are the mechanism through which liquids develop surface tension. Given that the volume force is proportional to $B_V \propto A$, the surface force should be $\sim (B_V)^{2/3}$ (as the surface $S \sim V^{2/3}$). Additionally, the term must be deducted from the volume term, and the coefficient a_s should have a magnitude similar to that of a_v. Indeed, as equals 13–18 *MeV*.

3. Coulomb term:

The term $-a_c Z(Z -1)A^{-1/3}$ is the third term, arises as a result of the Coulomb interaction between protons and is, of course, proportional to Z. This term is removed from the volume term because the Coulomb attraction makes a nucleus with a large number of protons less desirable (more energetic).

NOTES

The nucleus is simulated as a uniformly charged sphere to justify the form of the term and estimate the coefficient ac. Such a charge distribution's potential energy equals

$$E = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Q^2}{R}$$

Since we get the charge $q(r) = 4/3\pi r^3\rho$ = Q (³ from the uniform distribution inside the sphere, the potential energy is as follows:

$$E = \frac{1}{4\pi\epsilon_0}\int dq(\vec{r})\,\frac{q(\vec{r})}{|\vec{r}|} = \frac{1}{4\pi\epsilon_0}\int d^3\vec{r}\,\rho\frac{q(\vec{r})}{|\vec{r}|} = \frac{1}{4\pi\epsilon_0}\int_0^R dr\,4\pi r^2\rho\,\frac{q(r)}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\left(4\pi\int_0^R dr\,\frac{3Q}{4\pi R^3}r^2 Q\left(\frac{r}{R}\right)^3\frac{1}{r}\right) = \frac{1}{4\pi\epsilon_0}\int_0^R dr\,\frac{3Q^2r^4}{R^6} = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Q^2}{R}$$

Using the empirical radius formula $R = R_oA^{1/3}$ and the total charge formula $Q^2 = e^2 Z(Z-1)$ (indicating the fact that this term appears only when Z > 1, i.e., when there are at least two protons), we obtain:

$$\frac{Q^2}{R} = \frac{e^2Z(Z-1)}{R_0A^{1/3}}$$

It defines the Coulomb term's form. Then, using $a_c \approx$ with R_o = 1.25 *fm*, the constant a_c can be predicted to be 0.691 MeV, which is close to the experimental result.

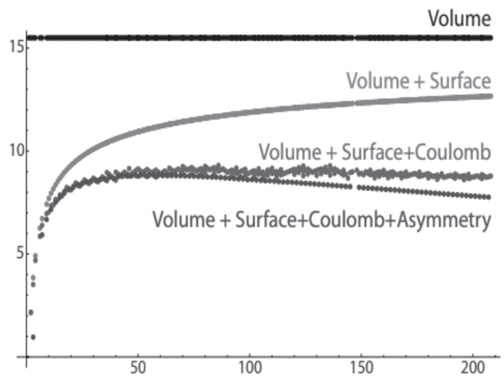


Fig. 1.8 *SEMF for stable nuclides*

We plot B (Z, A)/A vs. A. The various term contributions are added one by one to arrive at the final formula.

4. Symmetry term:

The Coulomb expression appears to imply that a nucleus with fewer protons and more neutrons would be desirable. This is not the case, therefore an alternative to the liquid-drop model must be utilized to account for the fact that stable nuclei have roughly equal numbers of neutrons and protons. Thus, the SEMF has a correction term that seeks to account for protons and neutrons' symmetry. This (and subsequent) correction can be described only by a more sophisticated model of the nucleus, the shell model, in conjunction with the quantum-mechanical exclusion principle, which will be discussed later. If more neutrons are added, they must be more energetic, increasing the overall energy of the nucleus. This gain more than compensates for the Coulomb repulsion, making it more favourable to have protons and neutrons in about equal amounts.

The symmetry term has a form of. Considering the fact that this term goes to zero for $A = 2Z$ and has a reduced influence for bigger A makes it easier to comprehend (while for smaller nuclei the symmetry effect is more important). The a_{sym} = 23MeV coefficient is given.

5. Pairing Term:

The final word refers to physical evidence indicating that similar nucleons tend to pair off. Then the binding energy is larger (δ > 0) if the nucleus is even-even, with all neutrons and protons paired-off. If a nucleus contains an odd number of neutrons and protons, it is advantageous to convert one of the protons to a neutron or vice versa (of course, taking into account the other constraints above). Thus, for odd-odd configurations, we must subtract (δ < 0) a term from the binding energy. Finally, for even-odd arrangements, this pairing energy (δ = 0) should have no effect. The term for pairing is then

$$+\delta a_p A^{-3/4} = \begin{cases} +a_p A^{-3/4} \text{ even-even} \\ 0 \text{ even-odd} \\ -a_p A^{-3/4} \text{ odd-odd} \end{cases}$$

with $a_p \approx$ 34MeV. [Sometimes the form $\propto A^{-1/2}$ is also found].

Isobaric Mass Parabola:

Some of the most essential aspects of nuclei's stability, such as the α-activity and stability properties of isobars, are explained by the binding energy formula. It is possible to derive semi-empirical formula for mass as

$$\mathrm{M(Z, A)} = \mathrm{Zm_p} + \mathrm{Am_n} - \mathrm{Zm_n} - \{a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z^2}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} \pm \delta(A,Z)\} \tag{1.51}$$

This equation can be expressed as follows:

$$M(Z,A) = \alpha A + \beta Z + \gamma Z^2 \pm \delta(A,Z) \tag{1.52}$$

Where, $\alpha = m_n - a_V + a_A + a_S / A^{1/3}$; $\beta = -4a_A - (m_n - m_p)$ and $\gamma = \frac{4aA}{A}$.

Equation (1.52) is quadratic in the *'Z'* axis. So the graph of *M(Z, A)* vs *Z* would seem like a parabola in shape.

NOTES

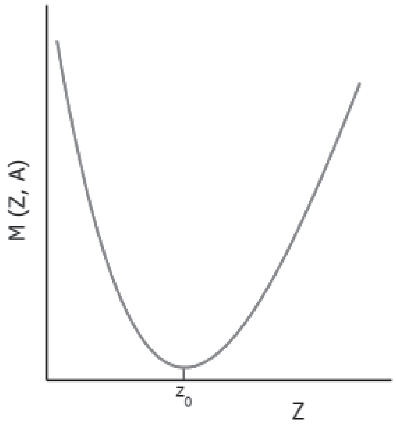


Fig. 1.9 *M(Z, A) vs Z parabolic curve.*

The nucleus at the bottom of the curve is the most stable in the series and has the highest binding energy. All isobars having a lower binding energy than the most stable one (at the bottom) will be found at the curve's arms. They will decay through electron or positron α-emission of electron (e^-) or positron (e^+) or *K*-capture. We have the minimum of the curve at $Z = Z_0$.

The condition for minimum is $\left.\frac{\partial M}{\partial Z}\right|_{A=constant} = 0$

From equation (1.52), $\frac{\partial M}{\partial Z} = \beta + 2\gamma Z$ we have

$$\text{At } Z = Z_0 \text{ ; } \frac{\partial M}{\partial Z} = 0$$

$$\text{So, } Z_0 = -\beta / 2\gamma \text{ or } \beta = -Z_0 2\gamma \quad (1.53)$$

Therefore, at Z_0; $M(Z, A) = \alpha A - \gamma Z_0^2 - \{\pm\delta(A, Z)\}$ (1.54)

We can have a variety of nuclear configurations depending on the number of protons and neutrons inside the nucleus, as shown in Table 1.3.

Table 1.3 *Different configurations of nuclei*

Z	N	A	Pairing energy $\delta(A,Z)$
Odd	Even	Odd	0
Even	Odd	Odd	0
Odd	Odd	Even	$-\delta_0$
Even	Even	Even	δ_0

During α-decay, the nucleus emits either an electron () or a positron (). The overall number of nucleons (*A*) remains constant, but the proton is converted to a neutron and vice versa, resulting in changes to *Z* and *N*. The reactions to the processes can be expressed as follows

$${}_Z^A X_N \xrightarrow{\beta^+ decay} {}_{Z-1}^{A} Y_{N+1} + \beta^+ + \nu + E_{\beta^+} \quad (1.55)$$

$${}_Z^A X_N \xrightarrow{\beta^- decay} {}_{Z+1}^{A} Y_{N\_1} + \beta^- + \bar{\nu} + E_{\beta^-} \quad (1.56)$$

NOTES

As a result of the decay, the neutrino (υ) and antineutrino ($\bar{\upsilon}$) and are produced. The released energy, and , should be positive. M(X) should be bigger than M(X) as a result (Y).

Odd *A* Isobars:

It is impossible to have an odd *Z* and an odd *N* in the same odd *A* isobar at the same time. The pairing energy, δ(A, Z), is zero in these alloys. As a result of solving equation (1.52), we arrive at

$$M(Z, A) = \alpha A + \beta Z + \gamma Z^2$$

By substituting value of α from equation (1.53), we get

$$\left.\begin{aligned} &M(Z, A) = \alpha A - 2\gamma Z_0 Z + \gamma Z^2 \\ &M(Z, A) = \gamma (Z - Z_0)^2 + \alpha A - \gamma Z_0^2 \\ &\text{On Changing Z to Z+1 or Z to Z-1, we get} \\ &M(Z+1, A) = \gamma (Z + 1 - Z_0)^2 + \alpha A - \gamma Z_0^2 \\ &M(Z-1, A) = \gamma (Z - 1 - Z_0)^2 + \alpha A - \gamma Z_0^2 \end{aligned}\right\} \quad (1.57)$$

During α^+ decay energy released is;

$$E_{\beta^+} = M(Z, A) - M(Z-1, A)$$

Therefore, using equation (1.57) we have

$$E_{\beta^+} = 2\gamma (Z - Z_0 - 1/2)$$

For α^- decay energy released given by;

$$E_{\beta^-} = M(Z, A) - M(Z+1, A)$$

$$E_{\beta^-} = 2\gamma (Z_0 - Z - 1/2)$$

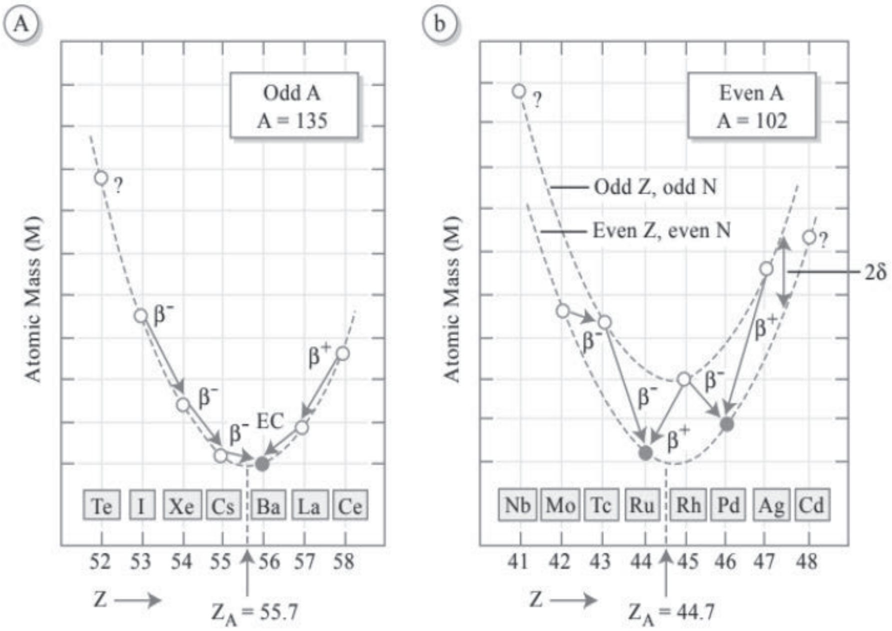
As binding energy is plotted against Z for a number of nuclei, it can be seen that the odd A is constant while the Z is variable.

According to Figure 1.10, this causes the formation of a parabola-like curve. (A). Using odd A series, the pairing energy is zero, and hence just one parabola is generated. The most stable isobar is located at the bottom. Electron emission is the process by which the isobars to the left of the most stable one deteriorates.

$$Te \xrightarrow{\beta-} I \xrightarrow{\beta-} Xe \xrightarrow{\beta-} Cs \xrightarrow{\beta-} Ba$$

Excess protons decay in isobars on the right side of the stable isobar via positron emission, K-capture, or a combination of the two.

$$Ce \xrightarrow{\beta+} La \xrightarrow{\beta+} Ba$$



(a) *(b)*

Fig. 1.10 *Isobaric mass parabola (a) odd A nuclei; (b) even A nuclei.*

Even *A* Isobars:

According to the odd-even effect, the result obtained for even A nuclei is different from the result obtained for odd A nuclei. In even A, the energy of pairing is not zero. Due to the fact that both odd-odd and even-even nuclei have an even A, they have two distinct pairing energies as indicated in the table above. As a result, it has two parabolas in its binding energy curve that are displaced by a factor of $2\delta_0$.

For even-even nuclei

$$M(Z,A) = \gamma(Z - Z_0)^2 + \alpha A - \gamma Z_0^{\ 2} - \delta_0 \qquad (1.58)$$

For odd-odd nuclei

$$M(Z,A) = \gamma(Z - Z_0)^2 + \alpha A - \gamma Z_0^{\ 2} + \delta_0 \qquad (1.59)$$

You may get *M(Z-1, A)* and *M(Z+1, A)* relations by altering *Z* to *Z-1* and *Z+1*, respectively. Even-even nuclei are more stable than odd-odd nuclei, as can be seen from equations 8 and 9. For α - decay of odd-odd nuclei

$$E_{\beta^-} = \underbrace{M(Z,A)}_{\text{odd-odd}} - \underbrace{M[(Z+1),A]}_{\text{even-even}}$$

We have obtained using equations (1.58) and (1.59);

$$E_{\beta^-} = 2\gamma[(Z_0 - Z) - 1/2] + 2\delta_0$$

For α^+ decay of odd-odd nuclei

$$E_{\beta^-} = 2\gamma[-(Z_0 - Z) - 1/2] - 2\delta_0$$

NOTES

Figure 1.10(b) depicts the isobaric mass parabola for even A nuclei. On the upper curve, odd-odd nuclei are unstable compared to even-odd nuclei, hence they must undergo α-decay to stabilize. The term 'Double α-decay' refers to nuclear processes in which two protons simultaneously become two neutrons or vice versa. For even-even nuclei, two or more stable nuclei are required. There are three stable nuclei in the A = 136 isobaric family.

1.8 MIRROR NUCLEI AND ISOTOPIC SPIN FORMALISM

We discovered that there can be several stable nuclei for a given A for even-A nuclei by minimizing the semi-empirical mass formula as a function of proton number (as illustrated in Figure 1.11 below).

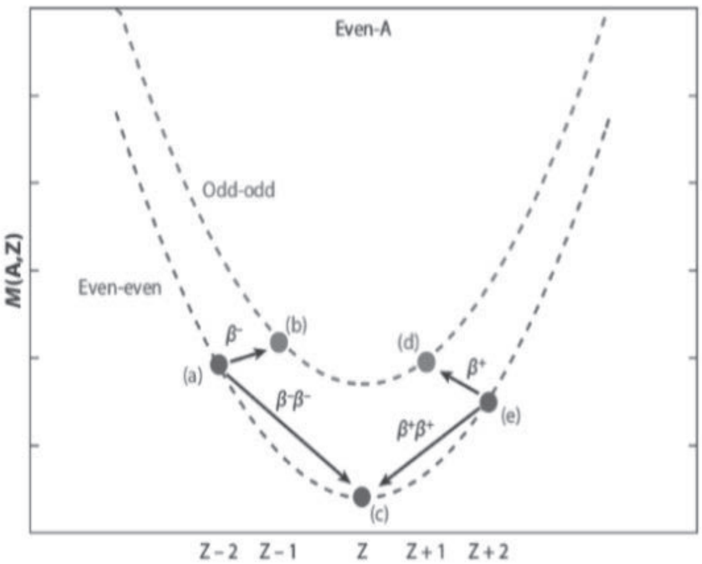


Fig. 1.11 *Mirror Nuclei and Isotopic Spin Formulation*

Mirror Nuclei:

The mirror nucleus is an atomic nucleus in which the protons and neutrons of one nucleus are swapped for those of the other. There are seven protons and eight neutrons in the nucleus of nitrogen-15 and eight protons and seven neutrons in the nucleus of oxygen-15. When a neutron is replaced by a proton in a nucleus, the isotopic nuclear force binding protons and neutrons remains approximately the same. In other words, or '*Mirror nuclei*' are pairs of nuclei whose proton number equals the neutron number in the other. Odd-A nuclei with one odd nucleon are the simplest instances. Charge $Z = (A + 1)/2$ and neutron number N are both $(A + 1)/2$ in one of the mirror nuclei; however, $Z = (A - 1)/2$ and $N = (A + 1)/2$ in the other. Examples are and or and . A proton is swapped for a neutron in each of these pairings of nuclei, which is the sole difference. Assuming that the binding energy of two mirror nuclei has the same *"nuclear component,"* it is likely that the force between nucleons does not differentiate between neutrons and protons. Thus, the difference in mass between two mirror nuclei can only be explained by the difference in proton and neutron masses and the varied Coulomb energies of the two. Here, the radius of the two nuclei may be determined (assumed to be the same).

Isotopic Spin Formulation

NOTES

To distinguish between protons and neutrons, **Werner Karl Heisenberg** created isospin in 1932, a German theoretical physics pioneer and one of the fundamental pioneers of quantum mechanics. To be clear, the isospin notion was first proposed in the 1950s, long before the quark model was developed in the 1960s.

A quantum number known as Isospin, I or I_3, is used to describe the strength of the nuclear force. To conserve isospin, substantial interactions degradation is required, according to a conservation law. isotopic spin was originally formed from this word, although the term "isobaric spin" is preferred by physicists because it is more exact.

As a result of these studies, it is clear that the strong interaction does not differentiate between these nucleons. The strong interaction between any two nucleons is identical regardless of whether they are neutrons or protons. Rather than treating protons and neutrons as distinct species, they are treated as various isospin states of the same fundamental nucleon particle in terms of strong interactions. The nucleon is the name given to this particle. Similarly, when only a strong nuclear force interacts with the three pions, π^0, π^+, and π^-, they appear to be three distinct states of the same particle. Isospin is comparable to spin mathematically, but has nothing to do with angular momentum. The spin word is included because isospins are added using the same rules as spin.

The concept 'Isospin' can now be introduced using an analogy. Because electrons have two spin values with regard to the z-direction, it is clear that this is the case. A non-uniform magnetic field in the z-direction can thus be used to identify $S_z = \pm$, i.e. As a result, in the absence of an external field, these two states of the same particle cannot be separated. Consequently, it is necessary to use superposition in order to describe the electronic spin state.

Electromagnetic interactions, on the other hand, allow us to distinguish between protons and neutrons. The strong interactions are also charge-independent. In nuclear physics, protons and neutrons cannot be distinguished as charged and neutral particles. As a result, these are simply two different states of the same particle (a nucleon). So, how will you be able to tell them apart? Isospin is the answer. The nucleon has this feature, which is theoretically equivalent to spin but has nothing to do with angular momentum, in an imagined space. Different values of the third component of this isospin, known as I_3 or I_z, have been assigned to the proton and neutron. In order to conserve isospin, this isospin has been linked to a conservation law that demands strong interaction decays. isotopic spin was originally formed from this word, although the term "isobaric spin" is preferred by physicists because it is more exact.

NOTES

Due to the fact that this third component can take on any value, we set $I_3 = \frac{1}{2}$ for the proton and $I_3 = \frac{1}{2}$ for the neutron. Thus, the nucleon has isospin $I = \frac{1}{2}$ in the same way as the electron has spin $s = \frac{1}{2}$, with the third component having two conceivable values.

Electric charge, Q, is related to the third component of the atom's isospin by

$$Q = I_3 + \frac{1}{2}$$

The explanation to why the proton is positively charged and the neutron is chargeless can be found by plugging in the values for the third component of isospin. Isospin multiplets can be found in a variety of different particles. As an example, three pions, π^+, π^0 and π^-, are all almost identical in mass and spin, but they all have slightly different properties. Despite the fact that they have various charges, they all respond in the same way when powerful interactions occur. So, the three states of pions are just three different ways of saying the same thing: pions. Their charges, on the other hand, cannot be described using the formula shown above. Because of this, the formula must be changed. It is now time to make a direct comparison between electrical and nucleonic properties.

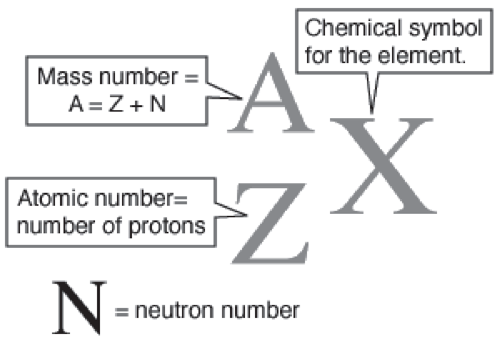
1.8.1 Basic Properties of Nuclei

Terminology:

A given atom is specified by the number of neutrons: N, protons: Z, electrons: there are Z electron in neutral atoms.

All atoms of any element have the same atomic number, Z. Although not all of them are the same there is a difference in the number of neutrons N between isotopes of the same element.

Isotopes are denoted by or more often by **or ^{A}X** where:



where X is the chemical symbol and A = Z + N is the mass number. For example: ${}^{12}_{6}C$, ${}^{13}_{6}C$, ${}^{14}_{6}C$.

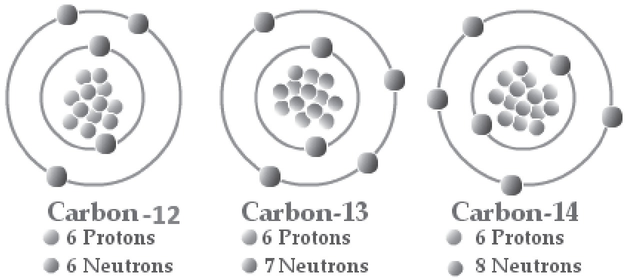


Fig. 1.12 *Isotopes of Carbon*

NOTES

When talking of different nuclei, one can refer to them as:

- Nuclide: atom/nucleus with a specific N and Z.
- Isobar: nuclides with same mass A (but ≠ Z, N).
- Isotone: nuclides with same N, but ≠ Z.
- Isomer: same nuclide (but different energy state).

Nuclear Radius

Since a nucleus cannot be described as a hard sphere with a specified radius, it is difficult to determine its radius. A practical specification of the range of nucleon densities that approximates our simple spherical model is still possible for many experimental situations (e.g., in scattering experiments). In order to calculate the radius of a nucleus, one must know the number of nucleons in the nucleus:

$$R = R_o A^{1/3}$$

Binding energy

It is one of the most significant experimental numbers in nuclear physics to measure the binding energy per nucleon (also written as BEN or B/A). which can be defined as;

$$m_N = Zm_p + Nm_n$$

The ionization energy of an electron in an atom is the average energy required to remove a single nucleon from a nucleus. When the BEN is really large, the nucleus is fairly stable. Nuclear scattering experiments are used to obtain an estimate of BEN. As the atomic number A increases, the binding energy per nucleon increases proportionately, as illustrated in Figure 1.13. According to many physicists, this graph is one of the most significant in all of physics. Two notes are required. BEN readings typically range between 6 and 10 MeV, with an average of around 8 MeV. To put it another way, while millions of electron volts are required to separate a nucleon from its normal nucleus, only 13.6 eV are required to ionize a single electron in the ground state of hydrogen. This is precisely why nuclear force is ∀ 'Strong.'

NOTES

The graph begins to fall at high A after ascending to a peak near iron (A = 56). According to their highest value, iron nuclei are the most stable in nature (it is also why nuclear fusion in the cores of stars ends with Fe). In the nucleus, two opposing forces are at work, which explains why the graph rises and then flattens. At low A values, the atomic forces between nucleons dominate the repulsive electrostatic forces between protons. On the other hand, high A values favour electrostatic forces, which prefer to separate the nucleus rather than maintain it together.

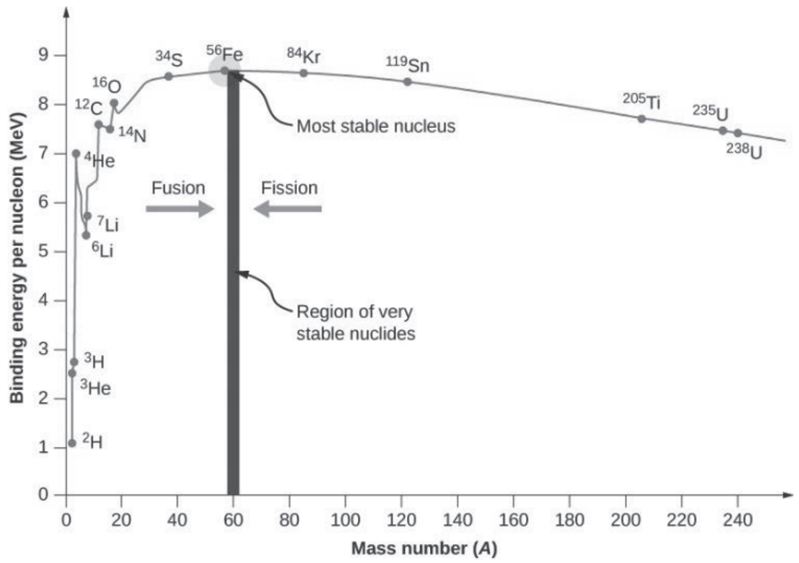


***Fig. 1.13** The BEN is largest for nuclei with a mass approaching ^{56}Fe*

As shown in this graph of binding energy per nucleon for stable nuclei. Thus, fusion of nuclei with masses less than or equal to those of Fe and fission of nuclei with masses larger than or equal to those of Fe are exothermic reactions.

The $m_N = Zm_p + Nm_n$ formula can be used to calculate the masses of all nuclides. But this has been shown empirically to be incorrect. From the special theory of relativity, it is known that each mass corresponds some energy *i.e.,* $E = mc^2$. Then, if you add up the masses of all the nuclei's parts, you'll get the total amount of energy they contain. There is a correlation between the mass of a nucleus and the energy it contains. There must be some additional energy needed to bind nuclei together, thus it seems sense that this isn't just the sum of its constituent energies. Having bound nuclei would be undesirable if the energy were equal, because all nuclei would be unstable, continually changing from their bound state to a mixture of protons and neutrons.

The binding energy of the nucleus is calculated using the difference in mass energy between the nucleus and its constituents. The binding energy ***B*** for the nucleus is calculated as follows:

$$B = [Zm_p + Nm_n - m_N(^AX)]c^2$$

This amount, however, should be described in terms of quantities

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that can be measured empirically. As a result, nuclear mass is expressed in terms of atomic mass, which is quantifiable $m_N(^AX)c^2 = [m_A(^AX)\text{-}Zm_e]c^2 + B_e$ where $m_A(^AX)$ is the *atomic mass* of the nucleus. Further neglect the electronic binding energy B_e by setting $m_N(^AX)c^2 = [m_A()\text{-}Zm_e]c^2$.

For the nuclear binding energy expression, use the following:

$$B = \{Zm_p + Nm_n - [m_A(^AX) - Zm_e]\}c^2$$

The neutron and proton separation energies are also qualities that are of interest:

$$S_n = B \;\; - B\,()$$

$$S_p = B \;\; - B\,()$$

In atomic physics, these are the valence nucleon energies, which are analogous to the ionization energies. These energies show the hallmarks of the nuclei's shell structure.

Check Your Progress

6. What do you mean by nuclear fission and fusion process?
7. Define the term nucleons.
8. What are the two types of departure in quadrupole moments of nuclei?
9. Write the semi-empirical mass formula.
10. What is mirror nuclei?

1.9 ANSWERS TO 'CHECK YOUR PROGRESS'

1. Rutherford and his collaborators performed scattering experiments with relatively higher energy α-particles and observed departure from Rutherford scattering formula at large angles.
2. Scattering of high energy electrons by nuclei constitutes the most direct method of measuring the charge radius of the nucleus and the nature of the nuclear charge distribution.
3. A large number of unstable fundamental particles, both charged and neutral, which are observed in nature (usually in the cosmic rays) or can be produced in the laboratory in high energy interactions. One of these is the μ-meson (previously called the μ-meson).
4. The mirror nucleus method is estimating the charge radius of a nucleus is based on the study of the energetics in the β^+ transformation of the mirror nuclei.

NOTES

5. The earliest method of estimating the potential radius was based on the study of alpha-disintegration of heavy nuclei like ^{238}U, ^{226}Ra, etc.
6. With the help of the binding fraction curve it is possible to explain in a qualitative manner the reasons for the α-disintegration of heavy nuclei as also of the energy release in nuclear fission and fusion processes.
7. A complex nucleus is made up of protons and neutrons, collectively known as nucleons.
8. The departures can occur due to two reasons :

   (a) Magnetic perturbations of the nearby levels;

   (b) Effect of the nuclear electric quadrupole moment.
9. The Semi-Empirical Mass Formula (SEMF) is,

   $M (Z, A) = Zm(^1H) + Nm_n - B (Z, A)/c^2$
10. The mirror nucleus is an atomic nucleus in which the protons and neutrons of one nucleus are swapped for those of the other. There are seven protons and eight neutrons in the nucleus of nitrogen-15 and eight protons and seven neutrons in the nucleus of oxygen-15. When a neutron is replaced by a proton in a nucleus, the isotopic nuclear force binding protons and neutrons remains approximately the same.

1.10 SUMMARY

- Rutherford's theory of a-particle scattering gives us an idea about the smallness of the nuclear size.
- Rutherford and his collaborators performed scattering experiments with relatively higher energy a-particles and observed departure from Rutherford scattering formula at large angles.
- Scattering of high energy electrons by nuclei constitutes the most direct method of measuring the charge radius of the nucleus and the nature of the nuclear charge distribution.
- A large number of unstable fundamental particles, both charged and neutral, which are observed in nature (usually in the cosmic rays) or can be produced in the laboratory in high energy interactions. One of these is the μ-meson (previously called the μ-meson).
- The mirror nucleus method is estimating the charge radius of a nucleus is based on the study of the energetics in the β^+ transformation of the mirror nuclei.
- A complex nucleus is made up of protons and neutrons, collectively known as nucleons.
- The mirror nucleus is an atomic nucleus in which the protons and neutrons of one nucleus are swapped for those of the other.

1.11 KEY TERMS

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- **Electron scattering experimental:** It refers to the scattering of high energy electrons by nuclei. It is the most direct method of measuring the charge radius of the nucleus and the nature of the nuclear charge distribution.
- **Mirror nucleus method:** Mirror nucleus method estimates the charge radius of a nuclelus which is based on the study of the energetics in the β^+ transformation of the mirror nuclei.
- **Nucleons:** A complex nucleus is made up of protons and neutrons collectively known as nucleons.
- **Mirror nucleus:** The mirror nucleus is an atomic nucleus in which the protons and neutrons of one nucleus are swapped for those of the other.

1.9 SELF-ASSESSMENT QUESTIONS AND EXERCISES

Short-Answer Questions

1. What do you mean by electron scattering?
2. State the muonic *x*- ray method.
3. Define binding energy curve.
4. What is nuclear spin?
5. How will you define magnetic and quadrupole moments of nuclei?
6. What are Schmidt lines?
7. Write semi-empirical mass formula.
8. Define mirror nuclei and isotopic spin formalism.

Long-Answer Questions

1. Explain the methods for determination of nuclear size and their interpretations.
2. Discuss binding energy curve for nuclei and its consequences with the help of examples.
3. What do you understand by the nuclear spin? Explain.
4. Explain the magnetic and quadrupole moments of nuclei with the help of examples.
5. Illustrate the semi-empirical mass formula and its application to mass parabolas.
6. Discuss the mirror nuclei and isotopic spin formalism.

1.13 FURTHER READING

NOTES

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UNIT 2 NUCLEAR FORCE AND TWO BODY PROBLEM

NOTES

Structure

2.0 Introduction
2.1 Objectives
2.2 Deuteron: Basic Properties
2.3 Existence of Excited States of Deuteron
2.4 n-p Scattering at Low Energies with Specific Square Well and Potential Results
2.5 Qualitative Discussion of the Effective Range Theory
2.6 Theory of p-p Scattering, Spin Dependence and Scattering Length
 2.6.1 Spin Dependence of n-p Interaction
 2.6.2 Scattering Length
 2.6.3 p-p Scattering at High Energy
2.7 Various Types of Two Body Nuclear Forces
2.8 Elementary Idea of Yukawa Theory of Nuclear Forces
2.9 Answers to 'Check Your Progress'
2.10 Summary
2.11 Key Terms
2.12 Self Assessment Questions and Exercises
2.13 Further Reading

2.0 INTRODUCTION

The nuclear forces are almost charge independent. If we assume they are, we can introduce a new quantum number which is conserved. For nucleons only, that is a proton and neutron, we can limit ourselves to two possible values which allow us to distinguish between the two particles. If we assign an isospin value of τ=1/2 for protons and neutrons (they belong to an isospin doublet, in the same way as we discuss the spin 1/2 multiplet), we can define the neutron to have isospin projection τ_z=+1/2 and a proton to have τ_z=−1/2. These assignments are the standard choices in low-energy nuclear physics. In particle physics, the opposite is the norm. This leads to the introduction of an additional quantum number called isospin. We can define a single-nucleon state function in terms of the quantum numbers n, j, m_j, l, s, τ and τz. In this unit, we will study in detail about the properties of deuteron, n-p and p-p scattering at low and high energies, spin interaction, scattering length and the elementary idea of Yukawa theory of nuclear forces.

2.1 OBJECTIVES

After going through this unit, you will be able to:

- Explain the properties of deuteron
- Describe n-p and p-p scattering at low and high energies
- Discuss the spin interaction and scattering length
- Describe the elementary idea of Yukawa theory of nuclear forces

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2.2 DEUTERON: BASIC PROPERTIES, BINDING ENERGY, SIZE, SPIN, MAGNETIC AND QUADRUPOLE MOMENTS

Deuteron is the only two-nucleon bound system made up of a proton and a neutron. The two other possible two-nucleon systems, the diproton (^{2}He) and the dineutron, do not exist as bound systems.

The following are the main experimentally determined properties of the deuteron:

1. The binding energy is

$$E_{Bd} = 2.2245 \pm 0.0002 \text{ MeV}$$

   The binding energy per nucleon in the deuteron is thus $f_{Bd} = 1.1122$ MeV. This is much smaller than the mean value of the binding fraction ($f_B = B/A$) for the nuclei with mass numbers of 4 or more. Even for the α-particle with $A = 4$, $f_B = 7.07$ MeV. Thus it is clear that the deuteron is a rather weakly bound structure, compared to most other nuclei.
2. The spin of the deuteron (total angular momentum) in the ground state in the unit of $\hbar$ is $I_d = 1$.
3. The measured magnetic moment of the deuteron in nuclear magneton (μ_N) unit is $\mu_d = 0.857414 \pm 0.000019$.
4. The deuteron also possesses a small but finite electric quadrupole moment which has the value $Q_d = +0.282 \text{ fm}^2 = 2.82 \times 10^{-31} \text{ m}^2$.
5. The parity of the deuteron ground state is even.

The observed values of the ground state spin and magnetic moment of the deuteron yield important information about the nature of this state.

The deuteron is made up of a proton and a neutron, both of which are spin 1/2 particles having the intrinsic magnetic moments $\mu_p = +2.7927$ and $\mu_n = -1.9131$ nuclear magnetons. Their sum is thus $\mu_p + \mu_n = 0.8796\mu_N$.

This value differs only slightly from μ_d given above. The difference is

$$\mu_p + \mu_n - \mu_d = 0.0222\mu_N$$

If we ignore this difference to a first approximation, then we may expect the proton and neutron magnetic moments to be antiparallel in the deuteron. Since μ_n is negative, the neutron magnetic moment is aligned antiparallel to its intrinsic spin s_n. Hence the proton and neutron spins s_p and s_n must be aligned parallel to each other in the deuteron, as shown in Fig. 2.1 giving the total intrinsic spin of the deuteron $s_d = s_p + s_n = 1$. This must be vectorially compounded with the relative orbital angular momentum L of the neutron-proton system to yield the correct value of the total angular momentum I_d of the deuteron which is its observed spin ($I_d = 1$). This can be done for the values of $L = 0$, 1 and 2 as shown in the vector diagrams in Fig. 2.2.

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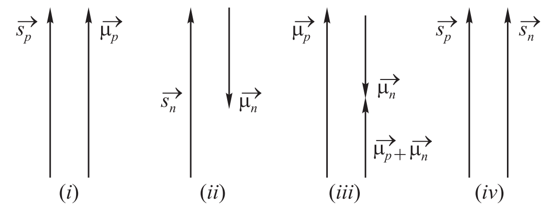

Fig. 2.1 *(i) and (ii) Relative orientations of the spins and magnetic moments of the neutron and the proton; (iii) Orientations of* μ_p *and* μ_n *in deuteron; (iv) Orientations of* s_p *and* s_n *in deuteron.*

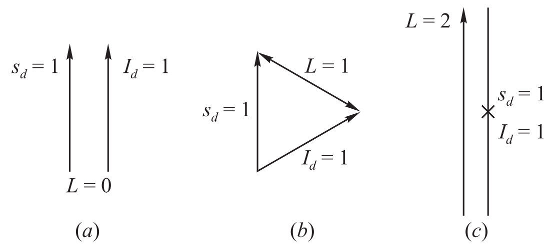

Fig. 2.2 *Vector addition of L and S to yield a value* $l_d = 1$ *for different values of L*

To the approximation in which the difference $\mu_p + \mu_n - \mu_d$ can be neglected, only the value $L = 0$ is admissible. The other two values $L = 1$ and 2 will contribute significantly to the magnetic moment due to the orbital rotation of the proton, which would introduce considerable difference between $\mu_p + \mu_n$ and μ_d.

The quantum mechanical theory of a system acted upon by a central force shows that the orbital angular momentum L is a constant of motion.

This means that each energy eigenstate of the system is characterized by a definite value of L. In particular, the ground state is an $L = 0$ state.

In what follows, we shall assume this to be true for the deuteron so that the potential of the interaction between the neutron and the proton is a function of r, the scalar distance between the two nucleons: $V = V(r)$.

As already stated, this is a very strong short-range interaction having the general appearance shown in Fig. 2.3.

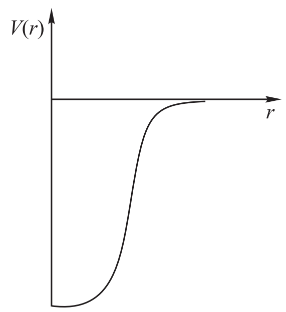

Fig. 2.3 *General form of the internucleon potential (central)*

Though the exact mathematical form of the interaction potential is not known, we can approximate such a strong short range, spherically symmetric potential by one of the following mathematical expressions:

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1. Rectangular potential well:

$$V = -V_0 \text{ for } r \leq b \qquad ...(2.1)$$
$$= 0 \text{ for } r > b$$

where $V_0 > 0$ and b is the radius of the well [see Fig. 2.4 (*a*)].

2. Exponential well [see Fig. 2.4 (*b*)]:

$$V = -V_0 \exp(-r/b) \qquad ...(2.2)$$

3. Yukawa well [see Fig. 17.4 (*c*)]:

$$V = -\{V_0/(r/b)\} \exp(-r/b) \qquad ...(2.3)$$

4. Woods-Saxon potential [Fig. 2.4 (*d*)]:

$$V = -V_0/\{1 + \exp[-(r - b)/c]\} \qquad ...(2.4)$$

where c is a constant representing the skin depth.

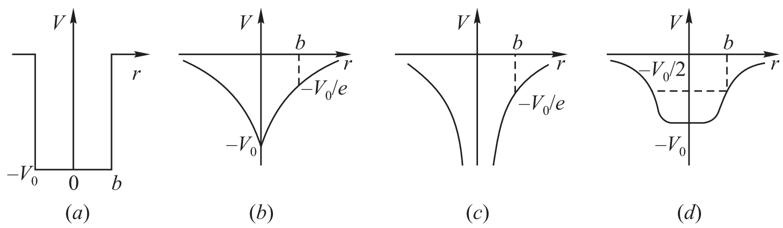

Fig. 2.4 *Two nucleon potentials. (a) Rectangular potential well; (b) Exponential well (c) Yukawa well; (d) Woods-Saxon potential well.*

Experimental evidences from proton-proton scattering shows that there is a repulsive core of the two-body potential at very short distances between the particles. Such a repulsive core potential may be either of finite height (soft core) or may be infinite (impenetrable or hard core). We shall consider these potentials in greater detail later.

2.3 EXISTENCE OF EXCITED STATES OF DEUTERON

We shall next investigate the possibility of the existence of an excited bound state of the deuteron. We shall consider two possibilities, *viz.*, the existence of a state with a higher value of n and that with a higher value of l.

Let us take the case of the $n = 1$ state for which a just bound state ($E_d = 0$), $k_0 b = 3\pi/2$ or for $b = 2$ fm

$$V_0 = \frac{9\pi^2\hbar^2}{4Mb^2} = 9V_{om} = 225 \text{ MeV}$$

This value of the potential depth disagrees violently from the $n - p$ potential depth calculated from the, deuteron ground state energy. For higher values of n, the disagreement would be still more violent.

So we conclude that no excited bound state of the deuteron exists with values of $n > 0$.

NOTES

We now consider the higher l values. For $l > 0$, the repulsive centrifugal potential tends to diminish the strength of binding of the deuteron. The effect increases with increasing value of l and is the least for $l = 1$. The magnitude of the centrifugal potential in this case at the boundary $r = b$ comes out to be about 21 MeV, which reduces the depth of the attractive potential to only 17 MeV which is much lower than that required to produce a just bound $n - p$ system (V_{om} = 25 MeV). The depth of the potential is reduced even more in the interior regions of the well ($r < b$). This qualitative reasoning shows that there cannot be any excited bound state of the deuteron for $l = 1$. These conclusions are confirmed by the solution of the radial wave equation with $l = 1$ which gives as solution the spherical Bessel function $J_{3/2}$ $(k_0 r)$ of order 3/2. Matching of the internal and external solutions at the boundary again leads to a contradiction with the theory for the bound state with l = 0.

2.4 N-P SCATTERING AT LOW ENERGIES WITH SPECIFIC SQUARE WELL AND POTENTIAL RESULTS

The relative motion of two particles of masses M_1 and M_2 can be described by the wave equation

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\,\psi = E\psi \;= E\,\psi \qquad ...(2.5)$$

where μ is the reduced mass. $E = E_L - E_C$ is the internal energy of the system. E_L is the energy in the L-system and E_C is the kinetic energy of the centre of mass given by

$$E_C = \frac{M_1}{M_1 + M_2} E_L \qquad ...(2.6)$$

For $n - p$ scattering $M_1 = M_2 = M$ (say) so that

$$E_C = E_L/2 \qquad ...(2.7)$$

So only half the laboratory energy is available for scattering in the C-system:

$$E = E_L - E_C = E_L/2 \qquad ...(2.8)$$

The angle of scattering θ_L in the L-systern is related to that in the C-system (θ_C) for $n - p$ scattering.

$$\theta_C = 2\theta_L \qquad ...(2.9)$$

Also the angle between the neutron and the proton after scattering in the L-system is always 90°.

Since the reduced mass of the n-p system is $\mu = M/2$, the wave Eq. (2.5) can be written as

$$\nabla^2\psi + \frac{M}{\hbar^2}\{E - V(r)\} = 0 \qquad ...(2.10)$$

Here $\psi = \psi\,(r, \theta, \phi)$; θ and ϕ are the centre of mass angles, r is the distance between the neutron and the proton. For scattering $E > 0$.

NOTES

Method of Partial Waves

The theoretical treatment of the neutron-proton scattering is based on the method of partial waves.

An incident plane wave can be expanded in terms of a series of spherical outgoing and spherical incoming waves in the limit of large r, as follows:

$$\psi_{\text{inc}} = \frac{1}{2ikr}\sum_{l=0}^{\infty}(2l+1)\,i^l\,[\exp\{i(kr - l\pi/2)\}$$

$$-\exp\{-i(kr - l\pi/2)\}\,P_l(\cos\theta) \qquad ...(2.11)$$

$P_l(\cos\theta)$ is the Legendre polynomial of order l; $k^2 = 2ME/\hbar^2$.

When a scatterer is present, the spherical outgoing waves in the above expression are affected, either in phase or in amplitude or in both. If only elastic scattering takes place (no reaction), then only the phase is affected. Since the scattered wave is a spherical outgoing wave with amplitude dependent on the angle of scattering we can write $\psi_{sc} = \{f(\theta)/r\}\exp(ikr)$. The total wave function in the presence of the scatterer is then

$$\psi(r) = \psi_{\text{inc}} + \psi_{\text{sc}} = \psi_{\text{inc}} + \frac{f(\theta)}{r}\exp(ikr) \qquad ...(2.12)$$

In analogy with Eq. (2.11), we can also write the total wave function as

$$\psi(r) = \frac{1}{2ikr}\sum_{l=0}^{\infty}(2l+1)\,i^l\,[\eta_l\exp\{i(ikr - l\pi/2)\}$$

$$-\exp\{-i(kr - l\pi/2)\}]\,P_l(\cos\theta) \qquad ...(2.13)$$

where $\eta_l = \exp(2i\delta_l)$ with δ_l real since only the phases and not the amplitudes of the outgoing waves arc affected. Obviously $|\eta_l|^2 = 1$.

In the scattering experiment, an incident beam of monoenergetic particles is scattered by an infinitely heavy scattering centre (in the C-system) at an angle θ. The differential scattering cross-section $\sigma(\theta)$ is then given by

$$\sigma(\theta) = |f(\theta)|^2 \qquad ...(2.14)$$

Comparing Eqs. (2.11) and (2.13), we get, using Eq. (2.12)

$$f(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)(\eta_l - 1)\,P_l(\cos\theta)$$

$$= \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\{\exp(2i\delta_l) - 1\}\,P_l(\cos\theta)$$

$$= \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\exp(i\delta_l)\sin\delta_l\,.\,P_l(\cos\theta) \qquad ...(2.15)$$

We then get from Eq. (2.14)

$$\sigma(\theta) = |f(\theta)|^2 = \frac{1}{k_2}\left|\sum_{l=0}^{\infty}(2l+1)\exp(i\delta_l)\sin\delta_l P_l(\cos\theta)\right|^2 \qquad ...(2.16)$$

The total scattering cross-section is obtained by integrating $\sigma(\theta)$ over the entire 4π solid angle:

$$\sigma_{tot} = \int \sigma(\theta)\, d\Omega = \int_0^{\pi} |f(\theta)|^2 . 2\pi \sin\theta\, d\theta$$

$$= \frac{2\pi}{k^2} \int_0^{\pi} \left| \sum_{l=0}^{\infty} (2l+1) \exp(i\delta_l) \sin\delta_l . P_l(\cos\theta) \right|^2 \sin\theta\, d\theta$$

Because of the orthogonality of the Legendre polynomials, we get finally

$$\sigma_{tot} = \frac{2\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)^2 \sin^2\delta_l \cdot \frac{2}{2l+1}$$

$$= \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2\delta_l \qquad ...(2.17)$$

Equation (2.15) shows that the scattering amplitude $f(\theta)$ can be expressed as a sum over the amplitudes $f_l(\theta)$ of scattering of the different partial waves

$$f(\theta) = \sum_{l=0}^{\infty} f_l(\theta) \qquad ...(2.18)$$

where

$$f_l(\theta) = \frac{2l+1}{2ik} \{\exp(2i\delta_l) - 1\}\, P_l(\cos\theta)$$

$$= \frac{2l+1}{k} \exp(i\delta_l) \sin\delta_l . P_l(\cos\theta) \qquad ...(2.19)$$

For calculating the cross-sections, the phase shifts δ_l must be known for different partial waves (see later). It may, at first sight, appear that the calculation of the cross-sections, using Eqs. (2.16) and (2.17) would be difficult, because of the large number of partial waves of different l involved. In practice, only a few l values are involved in these calculations; depending upon the energy (see below).

Limits of Energy for the Scattering of Different Partial Waves

Consider a neutron of energy E (in the C-system) and linear momentum p incident with an impact parameter q. Scattering will take place only if $q < b$. For $q > b$, the incident neutrons will not feel the interaction potential, which has a range b. Thus the largest impact parameter q, for which scattering occurs is $q_m = b$. For the neutrons having this impact parameter, the angular momentum will be $p \times q_m = p \times b$. According to quantum theory this can only be an integral multiple of $\hbar$.

Suppose for a given momentum, the product pb has a value such that

$$l\hbar \leq pb < (l+1)\,\hbar \qquad ...(2.20)$$

l is an integer. This means that the angular momentum of these particles will be $\hbar$. The maximum value p_{max} of the momentum for which this condition is satisfied is

$$p_{max}\, b = (l+1)\,\hbar$$

which gives

$$p_{max} = \frac{(l+1)\,\hbar}{b}$$

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and $$E_{max} = \frac{p_{max}^2}{M} = \frac{(l+1)^2 \hbar^2}{Mb^2} \quad ...(2.21)$$

So for *S*-wave *n-p* scattering (l = 0), the maximum energy in the *C*-system is

$$E_{max} = \frac{\hbar^2}{Mb^2} = 10 \text{ MeV}$$

We have seen before that for *n-p* scattering the energy in the *C*-system is half that in the *L*-system [Eq. (2.8)]. Hence the maximum energy for *S*-wave *n-p* scattering in the *L*-system is 20 MeV. To be on the safe side, the limiting values, are taken to be half those given above: E_{max} = 5 MeV in *C*-system and $(E_L)_{max}$ = 10 MeV for *S*-wave scattering.

We can divide the whole space surrounding the scattering centre into coaxial cylindrical zones with the axis parallel to the direction of the incident beam having radii λ, 2λ, 3λ etc. where $\bar{\lambda} = \hbar/p$, $\lambda = 2\pi\bar{\lambda}$ being the de Broglie wavelength.

Phase Shift

The above analysis is qualitative in nature and cannot be expected to give a correct quantitative picture. It is possible to determine the nature of variation of the phase shift with energy, under suitable approximations from the solutions of the radial equations with and without the scatter.

Making a separation of variables in the wave Eq. (2.10), we can get the radial equation in the presence of the scattering potential $V(r)$ as below (in the region $r < b$)

$$u_l'' + \left\{k^2 - U(r) - \frac{l(l+1)}{r^2}\right\} u_l = 0$$

where $u_l(r)$ is the radial function for the l^{th} partial wave and we have written

$$k^2 = \frac{M}{\hbar^2} E, U(r) = \frac{M}{\hbar^2} V(r)$$

When no scatterer is present, the radial equation becomes

$$v_l'' + \left\{k^2 - \frac{l(l+1)}{r^2}\right\} v_l = 0$$

The asymptotic solutions for the above equations are of the forms

$$u_l \sim \sin\left(kr - \frac{l\pi}{2} + \delta_l\right) \quad ...(2.22)$$

$$v_l \sim \sin\left(kr - \frac{l\pi}{2}\right) \quad ...(2.23)$$

The sign of δ_l determines whether the wave function $u_l(r)$ is ahead or lagging in phase with respect to the function $v_l(r)$.

For a negative (attractive) potential, $U(r) < 0$ and hence u_1'' has a larger negative value than v_l'' so that the wave function u_l (shown by the solid curve) has a greater curvature in the interior region ($r < b$) than v_l (shown by the dashed curve). This is illustrated in Fig. 2.5 (*a*) which shows that $\delta_l > 0$ for an attractive potential.

NOTES

Conversely for a repulsive potential. $\delta_l < 0$ as shown in Fig. 2.5 (*b*).

Quantitative estimate shows that for low energy scattering by a potential $V(r) \propto 1/r^n$. δ_l can be written as

$$\delta_l \sim k^{2l+1} \text{ for } 2l < n - 3$$

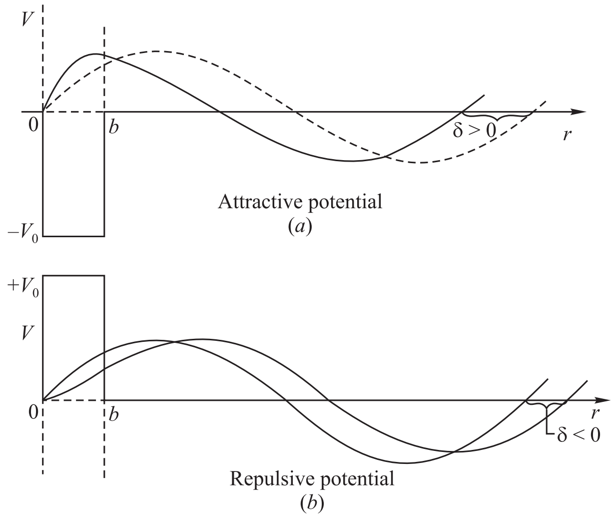

Fig. 2.5 *Nature of the radial solution for a given l for (a) attractive potential (δ_l > 0) and (b) repulsive potential (δ_l < 0).*

The measured phases δ_l ,completely determine the potential $V(r)$. However, the problem of determining the phases from the cross-sections is very complicated and has not been solved in practice for any case. For those cases in which only a small number of partial wave (l) are involved, fairly reliable information can some times be obtained about the phases.

Scattering Cross-section in the *L*-system

If we denote the differential scattering cross-sections in the C and L systems as $\sigma_C(\theta_C)$ and $\sigma_L(\theta_L)$, then it is possible to find a relationship between the two. Referring to Fig. 2.6, we note that since the number scattered into a given solid angle must be independent of the coordinate system chosen, we should write

Fig. 2.6 *Solid angles in C and L systems.*

$$\sigma_C(\theta_C)\, d\Omega_C = \sigma_L(\theta_L)\, d\Omega_L \quad \text{...(2.24)}$$

Since $\theta_C = 2\theta_L$, we have

$$d\Omega_C = 2\pi \sin\theta_C \, d\theta_C$$
$$= 8\pi \sin\theta_L \cos\theta_L \, d\theta_L$$

Also $$d\Omega_L = 2\pi \sin\theta_L \, d\theta_L$$

Hence we have

$$\sigma_C(\theta_C) = \frac{\sigma_L(\theta_L)}{4\cos\theta_L} \quad ...(2.25)$$

NOTES

It may be noted that the cross-section calculated above is in the *C*-system. However, the measurement of the cross-section is made in the *L*-system. The measured cross-sections $\sigma_L(\theta_L)$ must be transformed into $\sigma_C(\theta_C)$ with the help of Eq. (2.25) for comparison with theory.

Check Your Progress

1. What is deuteron?
2. How is an incident beam of monoenergetic particles scattered in the scattering experiment?

2.5 QUALITATIVE DISCUSSION OF THE EFFECTIVE RANGE THEORY

The low energy *n-p* scattering cross-section can be written as

$$\sigma = \frac{4\pi}{k^2 + 1/a_k^2}$$

where a_k is the scattering length for the energy E and $k^2 = (ME/\hbar^2)$. If we are able to express a_k as a function of k, then the energy dependence of the low energy scattering cross-section can be found.

Consider two energies E_1 and E_2 for which the wave equations are

$$u_1'' + \{k_1^2 - U(r)\}\, u_1 = 0 \quad ...(2.26a)$$

$$u_2'' + \{k_2^2 - U(r)\}\, u_2 = 0 \quad ...(2.26b)$$

where $k_1^2 = ME_1/\hbar^2$; $k_2^2 = ME_2/\hbar^2$; $U(r) = MV(r)/\hbar^2$. ...(2.27)

Multiplying the first of the Eqs. (2.26) by u_2'' and the second by u_1'', we get

$$u_2\, u_1'' + k_1^2\, u_1\, u_2 = U(r)\, u_1 u_2 \quad ...(2.28a)$$

$$u_1\, u_2'' + k_2^2\, u_1\, u_2 = U(r)\, u_1 u_2 \quad ...(2.28b)$$

Subtracting, we get

$$u_2\, u_1'' + u_1\, u_2'' = \frac{d}{dr}(u_2 u_1' - u_1 u_2') = (k_2^2 - k_1^2)\, u_1 u_2 \quad ...(2.29)$$

Integrating we get

$$\int_0^r \frac{d}{dr}(u_2 u_1' - u_1 u_2')\, dr = (k_2^2 - k_1^2)\int_0^r u_1 u_2\, dr \quad ...(2.30)$$

Since $u_1(0) = u_2(0) = 0$, we get

$$(u_2 u_1' - u_1 u_2')_r = (k_2^2 - k_1^2)\int_0^r u_1 u_2\, dt \quad ...(2.31)$$

Let us now consider the asymptotic solutions of the wave Eq. (2.26). Since $U(r) = 0$ for large r, the solutions are of the form:

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$$v_1(r) = \frac{\sin(k_1 r + \delta_1)}{\sin \delta_1},\; v_2(r) = \frac{\sin(k_2 r + \delta_2)}{\sin \delta_2} \quad ...(2.32)$$

v_1 and v_2 are normalized to unity at $r = 0$ so that $v_1(0) = 1$ and $v_2(0) = 1$.

We then get as before

$$v_2 v_1'' - v_1 v_2'' = \frac{d}{dr}(v_2 v_1' - v_1 v_2') = (k_2^2 - k_1^2)\, v_1 v_2 \quad ...(2.33)$$

Integration gives

$$(v_2 v_1' - v_1 v_2')_r - (v_2 v_1' - v_1 v_2')_0 = (k_2^2 - k_1^2) \int_0^r v_1 v_2 dr \quad ...(2.34)$$

From Eq. (2.32), we have

$$v_1' = \frac{k_1 \cos(k_1 r + \delta_1)}{\sin \delta_1},\; v_2' = \frac{k_2 \cos(k_2 r + \delta_2)}{\sin \delta_2} \quad ...(2.35)$$

Then $$v_1'(0) = k_1 \cot \delta_1,\; v_2'(0) = k_2 \cot \delta_2 \quad ...(2.35a)$$

We then get

$$(v_2 v_1' - v_1 v_2')_r - (k_1 \cot \delta_1 - k_2 \cot \delta_2) = (k_2^2 - k_1^2) \int_0^r v_1 v_2 dr \quad ...(2.36)$$

If now we push up the upper limits of integration in Eqs. (2.31) and (2.36) to $r = \propto$, we get

$$(u_2 u_1' - u_1 u_2')_\infty = (k_2^2 - k_1^2) \int_0^\infty u_1 u_2 dr \quad ...(2.37a)$$

$$(v_1 v_1' - v_1 v_2')_\infty - (k_1 \cot \delta_1 - k_2 \cot \delta_2) = (k_2^2 - k_1^2) \int_0^\infty v_1 v_2 dr \quad ...(2.37b)$$

At large r, $u_1(r)$ and $u_2(r)$ reduce to the asymptotic forms $v_1(r)$ and $v_2(r)$ respectively. So we get on subtraction of Eq. (2.37*a*) from Eq. (2.37*b*)

$$-(k_1 \cot \delta_1 - k_2 \cot \delta_2) = (k_2^2 - k_1^2) \int_0^\infty (v_1 v_2 - u_1 u_2)\, dr \quad ...(2.38)$$

Let us now choose the energy $E_1 = 0$ so that $k_1 = 0$. Also let $E_2 = E$ and $k_2 = k$. Then

$$\lim_{k_1 \to 0} (k_1 \cot \delta_1) = -\frac{1}{a}$$

where a is the Fermi scattering length. Further

$$k_2 \cot \delta_2 = k \cot \delta = -\frac{1}{a_k}$$

We then get

$$\frac{1}{a} - \frac{1}{a_k} = k^2 \int_0^\infty (v_0 v - u_0 u)\, dr$$

or, $$\frac{1}{a_k} = \frac{1}{a} - k^2 \int_0^\infty (v_0 v - u_0 u)\, dr \quad ...(2.39)$$

Now both $u(r)$ and $v(r)$ depend on E and hence on k. So the r.h.s. of Eq. (2.39) is in general energy dependent. Let us write

$$\int_0^{\infty} (v_0 v - u_0 u)\, dr = \frac{1}{2}\rho(0, E) \qquad \text{...(2.40)}$$

where $\rho(0, E)$ depends on the energies E_1 (= 0) and E_2 (= E). So we get

$$\frac{1}{a_k} = \frac{1}{a} - \frac{1}{2}\rho(0, E)\, k^2 \qquad \text{...(2.41)}$$

Since for small r, $V(r) > E$, the wave functions u_0 and v_0 for zero energy will differ very little from u and v respectively for the energy E so that for small r we can write $u(r) \approx u_0(r)$ and $v(r) \approx v_0(r)$. On the other hand, for large r (beyond the range of the n-p interaction potential), $u(r) \to v(r)$ and $u_0(r) \to v_0(r)$. So for some arbitrarily chosen range a of the potential we have

$$\int_0^{a} (v_0 v - u_0 u)\, dr = \int_0^{a} (v_0^2 - u_0^2)\, dr \qquad \text{...(2.42}a\text{)}$$

and

$$\int_a^{\infty} (v_0 v - u_0 u)\, dr = 0 \qquad \text{...(2.42}b\text{)}$$

So we get finally

$$\int_a^{\infty} (v_0 v - u_0 u)\, dr = \int_0^{\infty} (v_0^2 - u_0^2)\, dr = \frac{1}{2}\rho(0, 0) = \frac{1}{2}r_0 \qquad \text{...(2.43)}$$

where r_0 is a constant known as the effective range of the interaction which gives an average range of the interaction between the neutron and the proton. Thus from Eq. (2.42) we get

$$\frac{1}{a_k} = \frac{1}{a} - \frac{1}{2}r_0 k^2 \qquad \text{...(2.44)}$$

Equation (2.44) is deduced under *shape independent approximation*. It may be noted that the equations upto (2.41) are exact. The approximations are introduced in Eqs. (2.42) and (2.43). Actually these approximations are based on the assumptions of a strong short range interaction, the exact shape of which is not taken into account.

Equation (2.44) shows that if $1/a_k$ determined from low energy scattering data is plotted as a function of k^2 we get a straight line of slope equal to the effective range $r_0/2$. The intercept of this straight line with the ordinate gives the reciprocal of the Fermi scattering length ($1/a$).

The effective range r_0 depends on the range and depth of the interaction potential since u_0 in Eq. (2.43) obeys the wave Eq. (2.26) for $E = 0$. However, it does not depend on the shape of the potential.

Using Eq. (2.44), we can write the expression for low energy n-p scattering cross-section for triplet and singlet scattering as

$$\sigma_t = \frac{4\pi}{k^2 + \left(\frac{1}{a_t} - \frac{1}{2}r_{ot}k^2\right)^2} \qquad \text{...(2.45}a\text{)}$$

$$\sigma_s = \frac{4\pi}{k^2 + \left(\frac{1}{a_s} - \frac{1}{2} r_{os} k^2\right)^2} \quad ...(2.45b)$$

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where r_{ot} and r_{os} are the effective ranges for the triplet and singlet interactions.

The total cross-section for low energy *n-p* scattering is then

$$\sigma = \pi \left\{ \frac{3}{k^2 + \left(\frac{1}{a_t} - \frac{1}{2} r_{ot} k^2\right)^2} + \frac{1}{k^2 + \left(\frac{1}{a_s} - \frac{1}{2} r_{os} . k^2\right)^2} \right\} \quad ...(2.46)$$

This formula has been applied to analyze the scattering data upto the neutron energy of 10 MeV. The four parameters a_t, a_s r_{ot} and r_{os} required for the analysis of low energy data have been determined accurately in various experiments.

Effective Range Theory for the Bound Case

The effective range theory can be applied, even if one of the states is a bound state, as for the deuteron ground state. In this case

$$k^2 = \frac{M}{\hbar^2} E = \frac{M}{\hbar^2}(-\varepsilon_d) = -\alpha^2 \quad ...(2.47a)$$

Hence $$k = i\alpha = i\sqrt{M\varepsilon_d/\hbar^2} \quad ...(2.47b)$$

The wave equation for the outside region then becomes

$$u''_{\text{out}} + k^2 u_{\text{out}} = u''_{\text{out}} - \alpha^2 u_{\text{out}} = 0$$

With $\eta_0 = \delta$,

$$u_{\text{out}} = B\left\{\frac{\exp[i(kr+\delta)] - \exp[-i(kr+\delta)]}{2i}\right\}$$

$$= \frac{B}{2i}\{\exp(i\delta)\exp(-\alpha r) - \exp(-i\delta)\exp(\alpha r)\} \quad ...(2.48)$$

The coefficient of exp (αr) must be zero for the bound state. So exp ($-i\delta$) = 0 which means that ($i\delta$) must be a large real positive number. This would make B exp ($i\delta$) to be large, unless B is made small, so that the product B exp ($i\delta$) remains finite. We get in this case cot $\delta = i$ and

$$k \cot \delta = ik = -\alpha \quad (2.49)$$

Eq. (2.38) with $k_1 = 0$ and $k_2 = k = i\alpha$ then gives

$$\frac{1}{a} + k\cot\delta = \frac{1}{a} - \alpha = -\alpha^2 \int_0^\infty (v_0 v - u_0 u)\, dr \quad ...(2.50)$$

In the shape independent approximation, this reduces to

$$\alpha = \frac{1}{a} + \frac{1}{2} r_0 \alpha^2 \quad ...(2.51)$$

To a first approximation, if we neglect $\frac{1}{2} r_0 \alpha^2$ in Eq. (2.51) and $\frac{1}{2} r_0 k^2$ in Eq. (2.44), we can write $\alpha \approx \frac{1}{a_k} \approx \frac{1}{a}$ which gives the total (triplet) scattering cross-section as

$$\sigma_t = \frac{4\pi}{k^2 + \alpha^2}$$

2.6 THEORY OF P-P SCATTERING

The stability of nuclei containing protons and neutrons shows that strong short range attractive nuclear force must be present between protons at close distance, just like the neutron-proton force. This force is in addition to the long range Coulomb repulsive force between the protons and actually dominates over the latter within the nucleus.

Since no stable bound state of two protons (diproton or ^{2}He) is observed, the only means of investigating the *p*-*p* force is through experiments on proton-proton scattering. The *p*-*p* scattering experiments are easier to perform and interpret due to the following reasons:

(*a*) Monoenergetic proton beams at different energies are readily available from different kinds of accelerators. They are also more easily collimated.

(*b*) Protons are easier to detect and their energies can be measured more easily.

(c) Protons undergo both Coulomb and nuclear scattering. Interference between the waves scattered by the two types of force permits the determination of the sign of the phase shift for *p*-*p* scattering due to nuclear force more easily.

From the theoretical stand point the *p*-*p* scattering calculations are more complicated for the following reasons:

(a) The presence of Coulomb scattering in addition to nuclear scattering introduces interference effect. The Coulomb scattering calculations require special wave mechanical treatment because of the slow variation of the potential with distance.

(b) Effect of indistinguishability of the particles.

Partial wave method for the calculation of scattering cross-section is applicable only when the potential of the interaction is of the form $V(r) \sim 1/r^n$ with $n > 1$. Since the Coulomb potential $V \propto 1/r$ is not possible to apply the method of partial waves in this case.

Coulomb scattering calculations are made using parabolic coordinates.

The asymptotic wave function in the presence of Coulomb interaction is

$$\psi(r) = \exp\{ikz + i\eta \ln k(r - z)\} + \frac{f_c(\theta)}{r} \exp\{i(kr - \eta \ln 2kr + 2\eta_0 + \pi)\} \quad ...(2.52)$$

$$\text{Here } \eta = \frac{e^2}{4\pi\varepsilon_0 \hbar v}, \quad k = \frac{\sqrt{ME}}{\hbar} = \frac{Mv}{2\hbar}, \ \eta_0 = \arg \Gamma(1 + i\eta) \quad ...(2.53)$$

The first term in Eq. (2.52) represents the incident wave which is an almost plane wave slightly distorted due to the long range Coulomb potential. The second term is an almost spherical outgoing wave, slightly distorted due to the Coulomb potential.

$$f_c(\theta) = \frac{\eta}{2k \sin^2 \theta/2} \exp\{-i\eta \ln \sin^2 \theta/2\} \quad ...(2.54)$$

NOTES

$$\sigma_c(\theta) = |f_c(\theta)|^2 \qquad ...(2.55)$$

f_c (θ) is the Coulomb scattering amplitude. σ_c (θ) is the differential cross-section for Coulomb scattering in *C*-system. Using Eqs. (2.53) and (2.54) we get

$$\sigma(\theta) = \left(\frac{e^2}{4\pi\varepsilon_0 M v^2}\right)^2 \frac{1}{\sin^4 \theta/2} \qquad ...(2.56)$$

This is nothing but Rutherford scattering formula applied in the case of proton-proton scattering. Rutherford obtained it from classical considerations.

Since the two protons are identical (if we neglect spin) it is not possible for the detector *D* to distinguish between the incident proton scattered at θ and the recoil proton when the incident proton is scattered at (π – θ).

This is illustrated in Fig. 2.6.

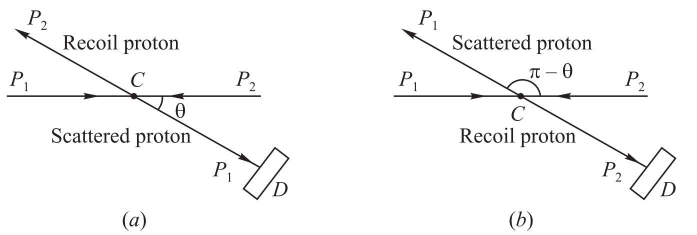

Fig. 2.6 *Detection of scattered and recoil protons by the same detector in p-p scattering.*

For the proton scattered at (π – θ) we write

$$\sigma(\pi - \theta) = \left(\frac{e^2}{4\pi\varepsilon_0 M v^2}\right)^2 \frac{1}{\cos^4 \theta/2} \qquad ...(2.56a)$$

So the cross-section is

$$\sigma_c(\theta) = \left(\frac{e^2}{4\pi\varepsilon_0 M v^2}\right)^2 \left(\frac{1}{\sin^4 \theta/2} + \frac{1}{\cos^4 \theta/2}\right) \qquad ...(2.57)$$

θ is the angle of scattering in the *C*-system. To get the cross-section in the *L*-system we have to use the relation $\theta_c = 2\theta_L$ and multiply the above equation by 4 cos θ_L.

Equation (2.57) is essentially classical. For wave mechanical treatment of the problem we have to take into account the exchange effect associated with the *indistinguishability* of the two protons and add up the amplitudes of the scattered waves at θ and (π – θ) and not the modulus squared of the amplitudes as was done above. The linear combination of *f*(θ) and *f*(π – θ) has to be properly symmetrized, remembering that the protons obey *F-D* statistics.

The space part of the wave function is *f*(θ) + *f*(π – θ) which is symmetric or *f*(θ) – *f*(π – θ) which is antisymmetric. The former has to be combined with the anti symmetric spin function, which is a singlet with *S* = 0 and a statistical weight 2*S* + 1 = 1. The latter has to be combined with the

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symmetric spin function, which is a triplet with $S = 1$ and has a statistical weight $2S + 1 = 3$. Using Eq. (2.54) we write

$$f_c(\theta) = \frac{\eta}{2k\sin^2\theta/2}\exp(-i\eta\ln\sin^2\theta/2)$$

$$f_c(\pi-\theta) = \frac{\eta}{2k\cos^2\theta/2}\exp(-i\eta\ln\cos^2\theta/2)$$

The scattering cross-section then becomes

$$\theta_c(\theta) = \frac{1}{4}\left|f_c(\theta) + f_c(\pi-\theta)\right|^2 + \frac{3}{4}\left|f_c(\theta) - f_c(\pi-\theta)\right|^2$$

$$= \left|f_c(\theta) + f_c(\pi-\theta)\right|^2 - \text{Re}\, f_c(\theta)\, f_c^*(\pi-\theta)$$

$$= \left(\frac{e^2}{4\pi\varepsilon_0 Mv^2}\right)^2\left[\frac{1}{\sin^4\theta/2} + \frac{1}{\cos^4\theta/2} - \frac{\cos\{\eta\ln\tan^2\theta/2\}}{\sin^2\theta/2\cos^2\theta/2}\right] \quad ...(2.58)$$

This is the Mott scattering formula.

For proton energies of 1 MeV and higher, $\eta = e^2/4\pi\varepsilon_0\hbar v$ is small so that the cosine in the last term in the above expression is almost unity unless $\theta = 0$ or π. So we get

$$\sigma_c(\theta) = \left(\frac{e^2}{4\pi\varepsilon_0 Mv^2}\right)^2\left[\frac{1}{\sin^4\theta/2} + \frac{1}{\cos^4\theta/2} - \frac{1}{\sin^2\theta/2\cos^2\theta/2}\right] \quad ...(2.59)$$

Equation (2.59) shows that the Mott formula gives a cross-section at the angle $\theta = 90^o$ in the C-system (45^o scattering angle in the L-system) which is half of what would be obtained if the effect of quantum mechanical indistinguishability is not taken into account (Eq. 2.57). This is in good agreement with observations at low energies.

Effect of Nuclear Force

When the results of *p-p* scattering experiments are compared with those calculated from the Mott formula given above, there is agreement only very low energies ($E < 0.1$ MeV). As the energy is increased, the experimental cross-sections are found to differ markedly from the theoretical values. For E upto about 0.6 MeV, the experimental values are lower, while for $E > 0.6$ MeV, they are higher than the theoretical values. Such disagreement indicates that nuclear potential between the two protons must be taken into consideration at higher energies, when the protons approach very close together.

It is reasonable to expect that the nuclear force between two protons has the same characteristics as that between a neutron and a proton, *i.e.*, it is a short range attractive force with a range of the same order of magnitude as the *n-p* force. So at low energies only $l = 0$ or S-scattering is expected to take place under the action of the nuclear potential.

If we write the wave function in terms of partial waves of different orders, then we have:

For pure Coulomb force

$$\psi(r) = \frac{1}{r}\sum_{l=0}^{\infty} v_l(r)\, P_l(\cos\theta)$$

$$= \frac{v_0}{r} + \frac{1}{r}\sum_{l=1}^{\infty} v_l(r)\, P_l(\cos\theta) \quad ...(2.60)$$

For Coulomb plus nuclear forces

$$\chi(r) = \frac{1}{r}\sum_{l=0}^{\infty} u_l(r)\, P_l(\cos\theta)$$

$$= \frac{u_0}{r} + \frac{1}{r}\sum_{l=1}^{\infty} u_l(r)\, P_l\,(\cos\theta) \quad ...(2.61)$$

Since for $l > 0$, the nuclear force has no effect, we must put $u_l(r) = v_l(r)$ for these higher l values. Hence we get

$$\chi(r) = \psi(r) + \frac{u_0(r) - v_0(r)}{r} \quad ...(2.62)$$

Assuming that for $l = 0$, the radial function $u_0\,(r)$ for the combined Coulomb plus nuclear force differs from $v_0(r)$ by a phase factor δ_0, it is possible to normalize the functions $u_0(r)$ and $v_0(r)$. It is then found that

$$\chi(r) = \exp\{ikz + i\eta \ln k\,(r - z)\}$$

$$+ \frac{g\,(\theta)}{r}\exp\{i\,(kr - \eta\ln 2kr + 2\eta_0 + \pi)\} \quad ...(2.63)$$

where

$$g(\theta) = \frac{e^2}{4\pi\varepsilon_0 Mv^2}\frac{\exp(-i\eta\ln\sin^2\theta/2)}{\sin^2\theta/2} + \frac{i}{2k}\{\exp(2_i\delta_0) - 1\} \quad ...(2.64)$$

As in the case of pure Coulomb scattering we have to make symmetric and antisymmetric combinations of the space functions $g(\theta)$ and $g(\pi - \theta)$ and multiply them by the appropriate statistical weights for spin orientations. We get

$$g_s\,(\theta) = g(\theta) + g\,(\pi - \theta)$$

$$= \left(\frac{e^2}{4\pi\varepsilon_0 Mv^2}\right)\left\{\frac{\exp(-i\eta\ln\sin^2\theta/2)}{\sin^2\theta/2} + \frac{\exp(-i\eta\ln\cos^2\theta/2)}{\cos^2\theta/2} + \frac{i}{k}[\exp(2i\delta_0) - 1]\right\} \quad ...(2.65)$$

$$g_a(\theta) = g(\theta) - g\,(\pi - \theta)$$

$$= \left(\frac{e^2}{4\pi\varepsilon_0 Mv^2}\right)\left\{\frac{\exp(-i\eta\ln\sin^2\theta/2)}{\sin^2\theta/2} - \frac{\exp(-i\eta\ln\cos^2\theta/2}{\cos^2\theta/2}\right\} \quad ...(2.66)$$

Notice that the nuclear force does not contribute to the antisymmetric function for which the minimum l is 1.

We get finally

$$\sigma(\theta) = \frac{1}{4}|g_s(\theta)|^2 + \frac{3}{4}|g_x(\theta)|^2$$

NOTES

$$= \left(\frac{e^2}{4\pi\varepsilon_0 Mv^2}\right)^2 \left[\frac{1}{\sin^4\theta/2} + \frac{1}{\cos^4\theta/2} - \frac{1}{\sin^2\theta/2\cos^2\theta/2}\right]$$

$$+ \frac{\sin^2\delta_0}{k^2} - \frac{e^2}{4\pi\varepsilon_0 Mv^2}\frac{\sin\delta_0\cos\delta_0}{k\sin^2\theta/2\cos^2\theta/2} \quad ...(2.67)$$

The first term in Eq. (2.67) is the Mott scattering formula for Coulomb force only. The second is the nuclear scattering term. The last represents the interference between the Coulomb and nuclear scattering. The sign of the last term depends on the sign of δ_0 and hence on the Fermi scattering length. So it is possible to determine the sign of δ_0 from the observed variation of $\sigma_{pp}(\theta)$ with θ and to decide whether there is a bound state of the *p-p* system or not. The results show that the *p-p* scattering length is negative so, that there cannot be any bound state of the *p-p* system. Further, the positive sign of δ_0 also shows that the *p-p* nuclear potential is attractive.

If a rectangular well is assumed for the nuclear part of the *p-p* potential, the following values of the depth and range are found: $(V_0)_{pp}$ = 13.3 MeV, b_{pp} = 2.58 fm. These values are in fair agreement with the corresponding values of the singlet *n-p* potential depth and range. It may be noted, that the spins of the protons must be antiparallel in the *S*-state to satisfy Pauli principle which results in $^1S_0$ state for the *p-p* system.

An effective range theory for proton-proton scattering can be developed to describe the variation of the *p-p* phase shift as a function of energy. In the shape-independent approximation, the formula corresponding to Eq. (2.44) is replaced by

$$C_0^2 k\cot\delta_0 + \frac{h(\eta)}{R} = -\frac{1}{a} + \frac{1}{2}r_{os}k^2 \quad ...(2.68)$$

where

$$C_0^2 = \frac{2\pi\eta}{\exp(2\pi\eta) - 1},\ \eta = \frac{e^2}{4\pi\varepsilon_0\hbar v} \quad ...(2.69)$$

$$R = \frac{4\pi\varepsilon_0\hbar^2}{Me^2} = 2.88\times10^{-14}\ \text{m} \quad ...(2.70)$$

$h(\eta)$ is a slowly varying function of energy, behaving logarithmically at high energies.

The first term in Eq. (2.68) is the value of $k\cot\delta$ which appears in the *n-p* scattering theory multiplied by the Coulomb penetration factor C_0^2. In the case of *n-p* scattering, the *S*-wave phase shift is derived from the observed scattering cross-section. But for *p-p* scattering, δ_0 is obtained from the angular distribution at a number of different energies. Equation (2.68) then yields the values of the *p-p* scattering parameters a_s and r_{os}. The *p-p* scattering length a_s is found to be negative, showing that the diproton has no bound state, as in the case of the *n-p* singlet ($^1S_0$) state.

Coulomb correction changes the value of a_s from –7.82 fm to about –17 fm which is comparable to the value –17.4 fm for *n-n* scattering, both of which are considerably less than the value of a_s = –23.7 fm for *n-p* scattering.

NOTES

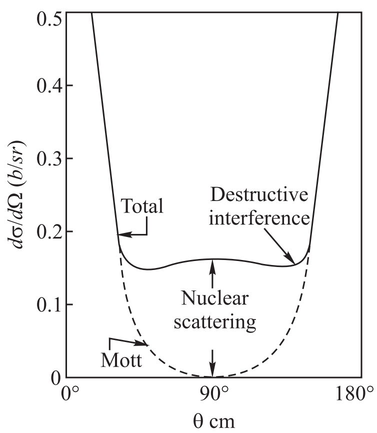

Fig. 2.7 *Angular distribution of p-p scattering in C-system at 2.5 MeV.*

Figure 2.7 shows the angular distribution of *p-p* scattering in *C*-system at the energy of 2.5 MeV which shows the effect of nuclear scattering as also the interference between Coulomb and nuclear scattering. As in the case of *n-p* scattering, low energy *p-p* scattering upto about 10 MeV can be accounted for by an *S*-wave ($l = 0$) interaction between the protons. Except in the forward and backward directions, the scattering is almost spherically symmetric. The interference between the Coulomb and the nuclear functions produce a minimum which moves to smaller angles (for $\theta < \pi/2$) at higher energies. The flat portion is mostly due to the nuclear forces. The effect of Coulomb scattering is confined to small angles.

2.6.1 Spin Dependence of n-p Interaction

In the previous section, we calculated the zero energy *n-p* cross-section σ_0, assuming a rectangular potential well, which was found to be about 3 barns. This is widely different from the experimental value of zero energy incoherent scattering cross-section $\sigma_0 = 20.38$ barns. To explain this wide discrepancy, E.P. Wigner assumed that the *n-p* interaction potential is dependent on the relative spin orientations of the neutron and the proton.

For parallel spins of the neutron and the proton (each of spin 1/2) as in the deuteron, the resultant spin angular momentum is $S = 1$, which, has a statistical weight $2S + 1 = 3$. For antiparallel spin orientations, the resultant spin angular momentum of the *n-p* system is $S = 0$ which has a statistical weight $2S + 1 = 1$.

If an unpolarized beam of neutrons is incident on a target containing protons with random orientations of spin, then some of the neutrons will be scattered by protons with parallel spins (triplet scattering) while the others will be scattered by protons with antiparallel spin orientations (singlet scattering). According to Wigner, the potentials of interaction in the two cases are different. Let us call them V_t and V_s. Both are strong, short-range attractive potentials and may be assumed to be rectangular potentials with the depths V_{ot} and V_{os} and ranges b_t and b_s respectively.

In scattering experiment, the relative probabilities of the occurrence of the triplet and singlet states are 3/4 and 1/4 respectively. Hence, the resultant cross-section of scattering of an unpolarized beam of neutrons from protons can be written as

$$\sigma = \frac{3}{4}\sigma_t + \frac{1}{4}\sigma_s \qquad ...(2.71)$$

NOTES

For zero energy neutrons, the triplet scattering cross-section can be written as $\sigma_t = 4\pi a_t^2$ where $a_t \approx +4.8$ fm is the triplet scattering length [see Eqs. (2.82) and (2.94)]. The plus sign in a_t is due to the fact that the *n-p* interaction potential in the triplet case can produce a bound state of the *n-p* system (the deuteron ground state).

Substituting the calculated value of $\sigma_t \approx 3b$ for zero energy, we get from Eq. (2.71)

$$20 = (3/4) \times 3 + (1/4)\,\sigma_s$$

or,

$$\sigma_s = 71 \text{ barns}$$

In analogy with the triplet case, we write

$$\sigma_s = 4\pi a_s^2 \qquad ...(2.72)$$

where a_s is the singlet Fermi scattering length. We can easily calculate a_s, using the relation

$$\frac{a_s^2}{a_t^2} = \frac{\sigma_s}{\sigma_t} = \frac{71}{3} = 23.7$$

so that

$$a_s/a_t = 4.86 \qquad ...(2.73)$$

This gives

$$a_s = 23.3 \text{ fm} \qquad ...(2.74)$$

The sign of the singlet scattering length can not be determined from the above considerations. If $a_s > 0$, then a singlet bound state of the neutron-proton system (*i.e.*, of the deuteron) may be expected to exist. However, its energy will be quite small, as is evident for the relatively large value of a_s. However, if $a_s < 0$, then the singlet state will not be a bound state.

2.6.2 Scattering Length

We have seen that at low energies (E_L < 10 MeV), *S*-wave scattering takes place, for which only $l = 0$ term has to be taken in the expression for $f(\theta)$.

$$f(\theta) = \frac{\exp(i\delta_0)}{k}\sin\delta_0 \qquad ...(2.75)$$

The differential scattering cross-section is

$$\sigma(\theta) = |f(\theta)|^2 = \frac{\sin^2\delta_0}{k^2} \qquad ...(2.76)$$

Thus $\sigma(\theta)$ is independent of θ so that the scattering is spherically symmetric in the *C*-system at low energies. The total cross-section is

$$\sigma_{\text{tot}} = \int \sigma(\theta)\, d\Omega = \frac{4\pi \sin^2\delta_0}{k^2} \qquad ...(2.77)$$

Equation (2.77) can be transformed as follows:

$$\sigma_{\text{tot}} = \int \sigma(\theta)\, d\Omega = \frac{4\pi}{k^2 \text{cosec}^2\delta_0} = \frac{4\pi}{k^2 + k^2\cot^2\delta_0} \qquad ...(2.78)$$

As the energy decreases, k also decreases, which means that both $\sigma(\theta)$ and σ_{tot} increases. Equations (2.76) and (2.77) would make $\alpha(\theta)$ and σ_{tot} go to infinity in the limit of zero energy ($k \to 0$). However, that would make the number of scattered particles infinitely large, which is impossible. Hence we stipulate that the limiting value of $k \cot \delta_0$ in the denominator of Eq. (2.78) should remain finite as $k \to 0$. If we write

$$\frac{1}{a_k} = -k \cot \delta_0 \qquad ...(2.79)$$

then we get from Eq. (2.78)

$$\sigma_{tot} = \frac{4\pi}{k^2 + 1/a_k^2} \qquad ...(2.80)$$

a_k is known as the scattering length for the energy E.

In order that σ_{tot} may remain finite as $k \to 0$ we stipulate that

$$\lim_{k\to 0} k \cot \delta_0 = \lim_{k\to 0}\left(-\frac{1}{a_k}\right) = -\frac{1}{a} \qquad ...(2.81)$$

where a is finite and is known as the *Fermi Scattering length*. We then get for zero energy neutrons

$$\lim_{k\to 0} k \cot \delta_0 = 4\pi\, a^2 = \sigma_0 \text{ (say)} \qquad ...(2.82)$$

Equation (2.75) shows that the scattered amplitude $f(\theta)$ remains finite as $k \to 0$ if δ_0 also goes to zero in this limit; *i.e.*, $\lim_{k\to 0} \delta_0 = 0$. Since $\sin \delta_0 = \delta_0$ as $\delta_0 \to 0$ we get in this case

$$\lim_{k\to 0} f(\theta) = \lim_{k\to 0} \frac{\delta_0}{k} = -a \text{ (say)} \qquad ...(2.83)$$

The amplitude of the scattered wave in the limit of zero energy neutrons is thus equal to the negative of the Fermi scattering length.

We can give a simple geometrical interpretation of the Fermi scattering length a. If we draw a sphere of radius $2a$, then the area presented by this sphere to a parallel beam of neutrons incident on it is $\pi(2a)^2$ or $4\pi a^2$ as shown in Fig. 2.8. So we interpret the Fermi scattering length as being equal to half the radius of an impenetrable sphere, such that all neutrons of zero energy intercepted by it are elastically scattered.

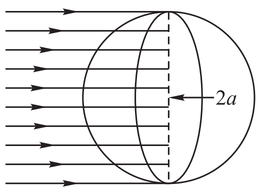

Fig. 2.8 *Geometrical inter-pretation of Fermi scattering length.*

Solution of the Radial Equation at Low Energy

The wave equation (2.10) in the presence of a scatterer can be split up into a radial part and an angular part by the usual method of separation of variables. The resulting radial equation for $l = 0$ becomes

$$\frac{d^2u}{dr^2} + \frac{M}{\hbar^2}\{E - V(r)\}u = 0 \qquad ...(2.84)$$

NOTES

Here $E > 0$. If we assume a rectangular potential well [Eq. (2.1)], the above equation reduces to

$$u''_{\text{in}} + k_2^2 u_{\text{in}} = 0 \qquad ...(2.85a)$$

$$u''_{\text{out}} + k^2 u_{\text{out}} = 0 \qquad ...(2.85b)$$

Here we have written $u_{\text{in}}(r)$ and $u_{\text{out}}(r)$ to be the radial functions inside and outside the well. We have also written

$$k_2^2 = \frac{M}{\hbar^2}(E + V_0),\ k^2 = \frac{M}{\hbar^2}E \qquad ...(2.86)$$

The solutions are

$$u_{\text{in}} = A \sin k_2 r \qquad ...(2.87a)$$

$$u_{\text{out}} = B \sin (kr + \eta_0) \qquad ...(2.87b)$$

where η_0 is the phase of the external function.

We can also find the nature of the radial function $u(r)$ at large r (asymptotic form) both with and without the scatterer from the expansions (2.22) and (2.23) respectively for $l = 0$ partial waves. These are as follows:

Without scatterer: $v_0(r) \sim \frac{\sin kr}{k}$ ...(2.88a)

With scatterer: $v_0(r) \sim \frac{\sin (kr + \delta_0)}{k}$ ...(2.88b)

Comparing the Eqs. (2.87b) and (2.88b), we then find the phase of the external wave to be $\eta_0 = \delta_0$.

Applying the boundary conditions at $r = b$, we get

$$k_2 \cot k_2 b = k \cot (kb + \delta_0) \qquad ...(2.89)$$

We then get after some simplification

$$\sin \delta_0 = \frac{\sin kb\,(k \cot kb - k_2 \cot k_2 b)}{\sqrt{k^2 + k_2^2 \cot^2 k_2 b}} \qquad ...(2.90)$$

This gives δ_0 as a function of E, V_0 and b.

Solutions for Neutrons of Zero Energy

In this case $E = 0$ so that $k = 0$ and

$$k_2^{\,2} = k_0^2 = \frac{MV_0}{\hbar^2}$$

The radial equations are

$$u''_{\text{in}} + k_0^2 u_{\text{in}} = 0 \text{ for } r < b \qquad .(2.91a)$$

$$u''_{\text{out}} = 0 \text{ for } r > b \qquad ...(2.91b)$$

The solutions are

$$u_{\text{in}} = A \sin k_0 r \qquad ...(2.92a)$$

$$u_{out} = B\,(r - a') \qquad ...(2.92b)$$

But from Eq. (2.87*b*), we have in the zero energy limit (for which both *k* and δ_0 go to zero)

$$\lim_{k\to 0} u_{out} = \lim_{k\to 0} B\sin(kr + \delta_0)$$

$$= \lim_{k\to 0} B\,(kr + \delta_0)$$

$$= \lim_{k\to 0} Bk\,(r + \delta_0/k) = B'\,(r - a) \qquad ...(2.92c)$$

NOTES

where we have substituted $\lim_{k\to 0}\left(\frac{\delta_0}{k}\right) = -a$ from Eq. (2.83). Comparing Eqs. (2-92*b*) and (2.92*c*) we get $a' = a$ = Fermi scattering length. Thus the external solution for zero energy neutrons reduces to

$$u_{out} = B\,(r - a) \qquad ...(2.92d)$$

Here we have put $B = B'$.

Nature of the Wave Functions

At low neutron energies $E << V_0$ so that $k^2 << k_0^2$ which means that k_2^2 in Eq. (2.85) is only slightly greater than $k_0^2 : k_2^2 \gtrsim k_0^2$. We also have $k_0^2 \gtrsim k_1^2$. This follows from the fact that the deuteron binding energy $E_{Bd} << V_0$. From the nature of the solution in the deuteron ground-state we know that $k_0 b \gtrsim \pi/2$. Hence from what has been stated above, we have $k_2\, b \gtrsim \pi/2$. This means that as in the case of the deuteron, the portion of the $\sin k_2\, r$ graph for the internal solution (2.87*a*) contained in the range $0 < r < b$ is slightly greater than half a loop of the sine curve as shown in Fig. 2.9. Hence the graph has a negative slope at $r = b$ so that the external solution (2.87*b*) must also have a negative slope at this point. Also since $k << k_2$, the wavelength of the external solution $\lambda >> \lambda_2$, the wavelength of the internal solution. These features of the solutions for low energy *n*-*p* scattering are shown in Fig. 2.9.

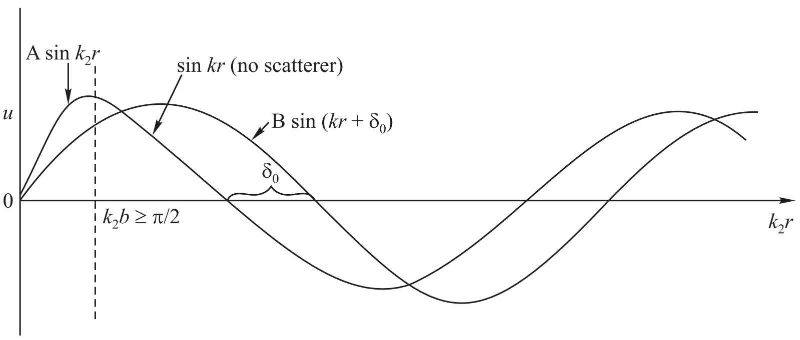

Fig. 2.9 *Wave functions of the scattered wave for 0 < r < b and for r > b*

If now we consider the case of zero energy of neutrons ($k = 0$) for which $k_2^2 = k_0^2$, the portion of the graph of the internal solution $\sin k_0\, r$ between $0 < r < b$ is again slightly larger than half a loop of the sine curve as shown in Fig. 2.10. This also has a negative slope at $r = b$ and hence the external solution which is the straight line given by Eq. (2.92*d*) has a negative slope which intersects the *r*-axis at $r = a > 0$. So the Fermi scattering length *a* is positive in this case.

NOTES

It is possible to calculate a from the boundary conditions applied to the solutions (2.92a) and (2.92d) at $r = b$:

$$(u'_{in}/u_{in})_b = (u'_{out}/u_{out})_b$$

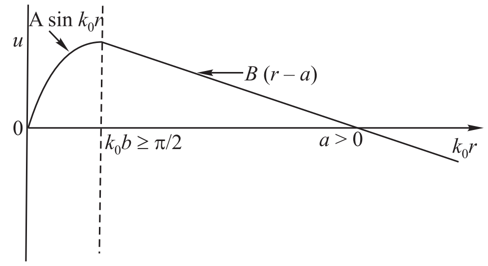

Fig. 2.10 *Nature of wave functions for zero energy neutrons. The outside function is a straight line with negative slope.*

This gives $k_0 \cot k_0\, b = \dfrac{1}{b-a}$ ...(2.93)

or, $a = b - \dfrac{\tan k_0 b}{k_0}$ ...(2.94)

For V_0 = 38 MeV and b = 2 fm, we get k_0 = 0.963 fm^{-1} so that $k_0\, b$ = 1.93 radians and tan $k_0\, b$ = –2.69. Hence we get a = 4.8 fm which gives $= 4\pi a^2 =$ 2.88 barns.

Sign of the Fermi Scattering Length

The discussions given above show that for an attractive n-p potential giving rise to bound state of the deuteron ($E = E_{Bd} < 0$) the Fermi scattering length a must be positive. This happens because the internal wave functions (u_{in}) both for the deuteron and for low energy ($E < V_0$) scattering including E = 0 have negative slopes at the boundary $r = b$; *i.e.*, both k_1b and k_0b are slightly greater than $\pi/2$. So the external linear function for zero energy neutron-proton scattering must intersect the r-axis at a point $r = a > 0$.

Let us now suppose that V_0 is gradually reduced so that k_0 is also reduced till k_0b becomes just equal to $\pi/2$. The slope of the internal function is then zero (cot k_0b = cot $\pi/2$ = 0) at $r = b$. The outside function for zero energy neutrons in this case is a straight line with zero slope, *i.e.*, it is parallel to the r-axis extending to infinity. This corresponds to the just bound case for which $a = \infty$.

If V_0 is reduced still further, $k_0\, b$ becomes smaller than $\pi/2$ so that the slope of the internal function is now positive (cot $k_0\, b > 0$) at $r = b$. The outside linear function for zero energy neutrons has thus a positive slope at $r = b$ and hence can intersect the r-axis only if it is produced backward. In this case $r = a < 0$ at the point of intersection *i.e.*, the Fermi scattering length for this unbound case is negative.

We thus conclude that fm a negative (attractive) n-p potential

1. For scattering of zero energy neutrons from a potential giving rise to a bound state of the n-p system, $a > 0$;

2. For scattering of zero energy neutrons from a potential giving rise to a just bound *n-p* system, $a = \infty$;

3. For scattering of zero energy neutrons from a weak attractive potential for which no bound state of the *n-p* system is possible, $a < 0$.

Thus from a knowledge of the sign of the scattering length it is possible to get an idea about the nature of a particular state of the *n-p* system, *i.e.*, whether it is bound or unbound.

2.6.3 p-p Scattering at High Energy

The experimental results on proton-proton scattering at high energies shown in Fig. 2.11 are very puzzling. For energies upto about 500 MeV, the angular distribution is flat except at small angles, showing that there is very little dependence of the differential cross-section on θ. Such spherically symmetric angular distribution is possible only for *S*-wave ($l = 0$) scattering which is clearly impossible at these high energies.

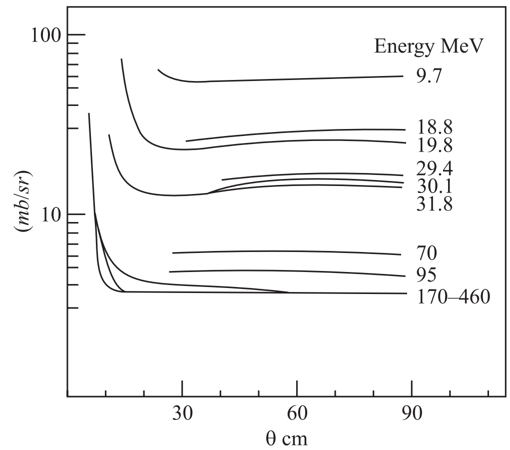

Fig. 2.11 *Variation of* σ_{pp} *(θ) with θ at high energies (experimental values).*

Estimates of the total cross-section also show puzzling features. While the *n-p* total cross-section shows $1/E$ variation the *p-p* total cross-section is almost constant between 150 MeV to 600 MeV, having a value of about 23 millibarns.

The hypothesis of no *p-p* force for values of l . θ is not tenable since the total calculated *S*-wave cross-section comes out to be 11 millibarns which is only about half the experimental value of 23 mb. The balance must be due to higher l values.

If it is assumed that only *S*, *P* and *D* waves are present then in the expression for $\sigma(\theta)$ there should be the following terms:

(*i*) A term proportional to $\sin^2 \delta$ due to ^1S scattering;

(*ii*) a term in $\sin^2 \delta_2 \, [P_2 (\cos\theta)]^2$ due to ^1D scattering;

(*iii*) a term in $\sin \delta_0 \sin \delta_2 \, P_2 (\cos\theta)$ due to interference between *S* and *D* scattering;

(*iv*) terms due to ^{3}P wave scattering with different phase shifts δ_{10}, δ_{11} and

δ_{12} for ${}^3P_0$ ${}^3P_1$ and ${}^3P_2$ substates due to tensor force;

(*v*) other interference terms proportional to products of first and third order spherical harmonics.

NOTES

The nearly isotropic angular distribution of *p-p* scattering can then be explained on the basis of the following two hypotheses: (*a*) a force exists which makes the coefficients of the *S* and *D* interference terms negative; (*b*) there exists a tensor force in the triplet states which builds-up the angular distribution in the middle region. Such mixture of forces to produce a flat distribution might be possible at one particular energy. But it is difficult to see how it is possible at all energies.

For this to happen, the *D*-scattering should become more important as the energy rises while the *S-D* interference term must change by the right amount to compensate this. Actually the coefficient of this interference term involving $\sin\delta_0 \sin\delta_1$ should be negative at high energy and should decrease as the energy rises (see Fig. 2.12).

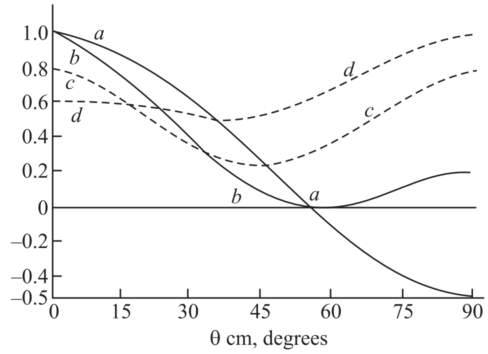

Fig. 2.12 *Contributions of S and D waves to p-p scattering.*

R. Jastrow has shown that a *p-p* force with a repulsive core would produce exactly these effects.

The whole space surrounding the scattering centre can be divided into a series of coaxial cylindrical zones of radii $\bar{\lambda}$, $2\bar{\lambda}$, $3\bar{\lambda}$ etc. where $\lambda = 2\pi\bar{\lambda}$ is the de Broglie wavelength given by $\lambda = h/\sqrt{2ME}$. If the range of interaction is *R* and the maximum energy E_m has a value which makes $\bar{\lambda}_m = R$, then only *S*-wave ($l = 0$) scattering takes place. On the other hand if the maximum energy is such that $\lambda_m = R/2$, then both *S* and *P* wave scatterings will take place.

Similarly if E_m is such that $\lambda_m = R/3$, then *S*, *P* and *D* wave scatterings will take place.

If the repulsive core range $c \sim 0.5$ fm, then the maximum energy for *S*-wave scattering is around 100 MeV. On the other hand the *P* and *D* wave scattering at this energy will take place due to the longer range attractive potential. Since repulsive potential gives negative phase shifts while attractive potential gives positive phase shifts, it is obvious that δ_0 is negative, while δ_2 is positive. So $\sin\delta_0 \sin\delta_2$ will be negative which is required to make the angular distribution flat even at high energies.

The energies at which the phase shift for a given l changes sign depends on the details of the potential shape. Using potential shape which agrees with experiment, the ^1S phase shift (δ_0) is found to become maximum between 10 to 20 MeV while it becomes negative at around 150 MeV. Thus the observed flat angular distribution between 150 to 500 MeV can be explained in terms of a uniformly changing phase shift in a properly chosen potential with a repulsive core.

It should be emphasized that the repulsive core is postulated for the proton-proton system only which is a singlet state.

2.7 VARIOUS TYPES OF TWO BODY NUCLEAR FORCES

The primary characteristics of the internucleon force can be inferred from a study of the ground state properties of the simplest bound state of the two nucleon system, which is the deuteron. In addition, internucleon scattering experiments at different energies also throw considerable light on the nature of the internucleon interaction.

Wave Equation for the Deuteron and its Solution

The Schrödinger wave equation for the deuteron in the C-system can be written as

$$\nabla^2\psi + \frac{2\mu}{\hbar^2}\{E - V(t)\}\psi = 0 \quad ...(2.95)$$

where $\psi = \psi(r, \theta, \phi)$. We have taken the interaction potential to be spherically symmetrical, depending only on the separation r between the neutron and the proton and independent of the angles θ and ϕ.

μ is the reduced mass of the system of the two particles. If M_p and M_n denote the proton and neutron masses, then

$$\mu = \frac{M_p M_n}{M_p + M_n} \quad ...(2.96)$$

Since $M_n \approx M_p = M$ (say), we get

$$\mu = M/2 \quad ...(2.97)$$

where M is usually taken to be equal to the proton mass M_p. Separating the variables (r, θ ϕ) and writing

$$\psi(r, \theta\ \phi) = \sum_l R_l(r) Y_l^m(\theta, \phi)$$
$$= \sum_l \frac{u_l(r)}{r} Y_l^m(\theta, \phi)$$

where Y_l^m's are the normalized spherical harmonics, we get the radial equation as

$$\frac{d^2u_l}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\right]u_l = 0 \quad ...(2.98)$$

The term containing $l(l + 1)/r^2$ is known as the centrifugal potential which is added to the assumed neutron proton-potential $V(r)$.

NOTES

The above Eq. (2.98) has been solved with the different types of potential given in the previous section. The results are found not to depend much on the shape of the potential assumed. So we shall consider the solution for the rectangular potential well (Eq. 2.1) which is the simplest that can be handled mathematically. Equation (2.98) then becomes for $l = 0$ (ground state):

For $r < b$, $$\frac{d^2u_{\text{in}}}{dr^2} + \frac{2\mu}{\hbar^2}(E + V_0)u_{\text{in}} = 0 \qquad ...(2.99a)$$

For $r > b$, $$\frac{d^2u_{\text{out}}}{dr^2} + \frac{2\mu E}{\hbar^2}u_{\text{out}} = 0 \qquad ...(2.99b)$$

Here we have written u_{in} and u_{out} for the radial functions in the inside ($r < b$) and outside ($r > b$) regions of the potential well.

Since the deuteron ground state is a bound state, its energy is negative ($E = -E_d$).

So we get, using Eq. (2.97) and putting $E = -E_d$ (where $E_d > 0$)

$$\frac{d^2u_{\text{in}}}{dr^2} + \frac{M}{\hbar^2}(V_0 - E_d)u_{\text{in}} = 0 \qquad ...(2.100a)$$

$$\frac{d^2u_{\text{out}}}{dr^2} - \frac{M}{\hbar^2}E_d\, u_{\text{out}} = 0 \qquad ...(2.100b)$$

Writing $$k_1^2 = \frac{M}{\hbar^2}(V_0 - E_d) \qquad ...(2.101a)$$

and $$\alpha^2 = \frac{M}{\hbar^2}E_d \qquad ...(2.101b)$$

we get $$k_1^2 = k_0^2 - \alpha^2$$

where $$k_0^2 = \frac{M}{\hbar^2}V_0 \qquad ...(2.102)$$

The wave equations then become

$$u''_{\text{in}} + k_1^2\, u_{\text{in}} = 0 \qquad ...(2.103a)$$

$$u''_{\text{out}} - \alpha^2\, u_{\text{out}} = 0 \qquad ...(2.103b)$$

Here u'' has been written for $(d^2 u/dr^2)$.

The solutions of Eqs. (2.103) are of the forms:

$$u_{\text{in}} = A \sin k_1 r + A' \cos k_1 r \qquad ...(2.104a)$$

$$u_{\text{out}} = B \exp(-\alpha r) + B' \exp(\alpha r) \qquad ...(2.104b)$$

Since the radial function $R(r) = u(r)/r$, we must put $A' = 0$ in Eq. (2.104*a*) which would otherwise make $u_{\text{in}} \to \infty$ as $r \to 0$. Similarly we must put $B' = 0$ in Eq. (2.104*b*) which would otherwise make $u_{\text{out}} \to \infty$ as $r \to \infty$. Hence we get finally

$$u_{\text{in}} = A \sin k_1 r \qquad ...(2.105a)$$

$$u_{\text{out}} = B \exp(-\alpha r) \qquad ...(2.105b)$$

The two solutions given above must be matched at the boundary $r = b$ which requires that

$$\left(\frac{u'_{\text{in}}}{u_{\text{in}}}\right)_b = \left(\frac{u'_{\text{out}}}{u_{\text{out}}}\right)_b \qquad ...(2.106)$$

where we have written u' for (du/dr). This condition requires that

$$k_1 \cot k_1 b = -\alpha \qquad ...(2.107)$$

NOTES

Substituting for k_1 and α we then get the transcendental equation

$$\cot k_1 b = -\alpha/k_1 = -\sqrt{\frac{E_d}{V_0 - E_d}} \qquad ...(2.108)$$

This equation cannot be solved analytically but must be solved graphically. However, we can get an idea about the minimum depth V_{om} of the potential which will give a just bound $n - p$ system for which we must put $E_d = 0$. We then get $\alpha = 0$ and $k_1 = k_0 = \sqrt{MV_{om}/\hbar^2}$ so that the condition (2.108) reduces to

$$\cot k_0 b = 0 \qquad ...(2.109)$$

This gives

$$k_0 b = \sqrt{\frac{MV_{om}}{\hbar^2}}.b = \frac{\pi}{2} \qquad ...(2.110)$$

or,

$$V_{om} = \frac{\pi^2\hbar^2}{4Mb^2} = \frac{1.026\times 10^{-28}}{b^2} \text{ MeV} \qquad ...(2.111)$$

Since b is constant, Eq. (2.111) shows that the product $V_{om}b^2$ is a constant. The minimum depth V_{om} of the rectangular potential well, which would make the $n - p$ system just bound, can be calculated from Eq. (2.111), provided we make a suitable guess about the range b of the internucleon potential. For b = 2 fm = 2 × 10^{-15} m, which is the mean internucleon separation in the nuclei, we get V_{om} = 25 MeV.

In the actual case E_d = 2.226 MeV. The value of the minimum potential depth V_{om} given above shows that the deuteron binding energy $E_{Bd} << V_0$. Putting $\varepsilon_d = E_{Bd}$ in Eqs. (2.101), we have

$$k_1 = \frac{\sqrt{M(V_0 - \varepsilon_d)}}{\hbar} \text{ and } \alpha = \frac{\sqrt{M\varepsilon_d}}{\hbar} \qquad ...(2.112)$$

The transcendental Eq. (2.107) can be written as

$$\cot k_1 b = -\frac{\alpha b}{k_1 b} \qquad ...(2.113)$$

Writing $x = k_1 b$ we then have the two equations

$$y = \cot x \qquad ...(2.114a)$$

$$y = -\alpha b/x \qquad ...(2.114b)$$

Putting in the numerical values, we have

$$\alpha^2 = \frac{M\varepsilon_d}{\hbar^2} = 1.67\times 10^{-27} \times 2.226 \times 1.6 \times 10^{-13}/(1.054\times 10^{-34})^2$$

or,

$$\alpha = (2.44/1.054) \,.\, 10^{14} = 2.314 \times 10^{14}$$
$$= 0.2314 \text{ fm}^{-1}$$

Taking b = 2 fm, we then have from Eq. (2.114b), since αb = 0.463

$$y = -0.463/x \qquad ...(2.114c)$$

Equations (2.114a) and (2.114c) have to be solved graphically.

In Fig. 2.13, we have plotted the two graphs the points of intersection of which give the possible solutions.

NOTES

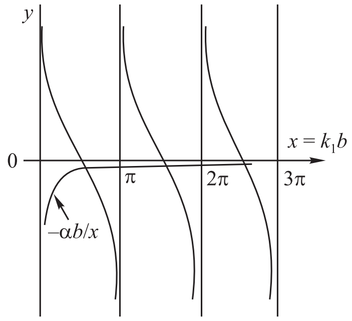

Fig. 2.13 *Graphical solution of the transcendental equation cot x = –αb/x where x = k_1b.*

As the figure shows the points of intersection of Eq. (2.114*c*) with the set of graphs $y = \cot x$ [Eq. (2.114*a*)] have small negative values. The successive values of k_1b must then be slightly greater ($\gtrsim$) than π/2, 3π/2, 5π/2 etc., or in general

$$k_1b \gtrsim (2n + 1)\,\pi/2 \qquad \text{...(2.115)}$$

where $n = 0, 1, 2, 3$, etc. The smallest of these values ($n = 0$) corresponds to the ground state for which $k_1b \gtrsim \pi/2$.

If we write $$k_1b = \frac{\pi}{2} + \varepsilon_0 \qquad \text{...(2.116)}$$

where ε_0 is a small number, we get

$$\cot k_1b = \cot\left(\frac{\pi}{2} + \varepsilon_0\right) = -\tan\varepsilon_0 \approx -\varepsilon_0 \qquad \text{...(2.117)}$$

Hence from Eq. (2.113), we have

$$k_1b \cot k_1b = -(\pi/2 + \varepsilon_0)\,\varepsilon_0 = -\alpha b = -0.463 \qquad \text{...(2.118)}$$

Neglecting ε_0^2 we then get

$$\varepsilon_0 \approx \frac{2\alpha b}{\pi} = \frac{2 \times 0.463}{\pi} = 0.295 \qquad \text{...(2.119)}$$

Hence $$k_1b = \frac{\pi}{2} + \varepsilon_0 = 1.87 \qquad \text{...(2.120)}$$

Putting $b = 2$ fm $= 2 \times 10^{-15}$ m, we then get, using Eq. (2.112)

$$V_0 - \varepsilon_d = 36 \text{ MeV}$$

or, $$V_0 = 38 \text{ MeV} \qquad \text{...(2.121)}$$

The relative magnitudes of the depth V_0 of the rectangular potential and the binding energy $E_{Bd} = \varepsilon_d$ of the deuteron in the ground state for the case $b = 2$ fm are shown in Fig. 2.14.

NOTES

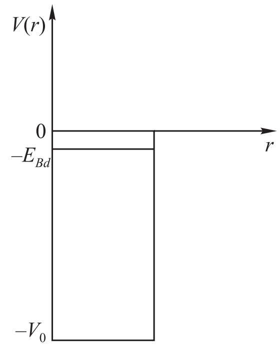

Fig. 2.14 *Deuteron ground state (ε_d) in the rectangular well of death 38 MeV.*

Check Your Progress

3. How are Coulomb scattering calculations made?
4. How can the primary characteristics of the internucleon force be inferred?

2.8 ELEMENTARY IDEA OF YUKAWA THEORY OF NUCLEAR FORCES

The electromagnetic interaction between two charged particles can be explained in terms of the interaction of these particles with the electromagnetic field. The electromagnetic field equations are

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\rho/\varepsilon_0 \qquad \text{...(2.122)}$$

$$\nabla^2 A - \frac{1}{C^2}\frac{\partial^2 A}{\partial t^2} = -\mu_0 j \qquad \text{...(2.123)}$$

where ρ and j are the charge and current densities. ϕ and A are the electric potential (scalar) and the magnetic vector potential respectively. The above equations actually constitute four scalar equations (one for ϕ and three for the three components of A) which can be combined to give a relativistically covariant from.

The static form of the equations is

$$\nabla^2\phi = -\rho/\varepsilon_0 \qquad \text{...(2.124)}$$

which has a solution

$$\phi(r) = -\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r')}{|r - r'|}d\tau' \qquad \text{...(2.125)}$$

For a point charge q located at $r' = r_1$, this gives

$$\phi(r) = \frac{q}{4\pi\varepsilon_0}\int\frac{\delta(r' - r_1)}{|r - r'|}d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{|r - r_1|} \qquad \text{...(2.126)}$$

NOTES

Here $\delta(r' - r_1)$ is the Dirac delta function.

If we take two point charges q at r_1 and r_2, the potential energy of q at r_2 in the field of q at r_1 is

$$V = q\phi(r_2) = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{|r_2 - r_1|} \quad ...(2.127)$$

Moving charges produce a radiation field that can be described in terms of photons which are the quanta of the e.m. field. The field energy is the sum of the energies of the quanta. In quantum electrodynamics, the e.m. field is considered jointly with the photons and the charges (sources). The photons are created and annihilated during emission and absorption respectively.

The interaction between two charges is described in terms of the emission of a photons by one charge and its absorption by the other.

The basic ideas underlying the quantum field theory can be extended to the case of nuclear interaction as was first done by the Japanese scientist H. Yukawa in 1935.

According to Yukawa the quantum of the nuclear field is a particle (charged or uncharged) of finite rest mass to account for the short range character of the nuclear interaction. As we shall see the particle should have a rest mass of about 300 m_e. Such a particle known as a π-meson was later discovered in the cosmic rays.

The mechanism of the internucleon interaction, in analogy with the e.m. interaction, may be visualized as follows. When two nucleons are within the range of internucleon interaction, they continually exchange a virtual meson, which can exist only for a very short time $\Delta t \sim \hbar/\Delta E$ determined by the uncertainty relation. Taking the velocity of the particle to be almost c we get $\Delta t \sim b/c$ where b is the range of the interaction. The mass of the virtual meson is given by

$$m = \frac{\Delta E}{c^2} = \frac{\hbar}{c^2 \Delta t} = \frac{\hbar}{bc} \quad ...(2.128)$$

Assuming the range $b \sim 2$ fm, we then get $m \sim 200\ m_e$ for the mass of the virtual meson.

The meson does not normally exist in the real state unless sufficient energy is available for its creation. The amount of energy required for this purpose is of the order of 100 MeV or more.

As stated above during the exchange between the two nucleons, the meson exists in the virtual state for a time $\Delta t = b/c = 10^{-23}$ s. During this time there is a temporary failure of the law of conservation of energy by an amount ΔE determined by the uncertainty relation which is of the order of 100 MeV. However, because of the very rapid exchange of the virtual meson, the violation of energy conservation may be regarded as compensated within the time Δt is conformity with the uncertainty relation.

For the creation of the meson in the real state it is necessary to supply sufficient energy (~ 100 MeV or more) which usually comes from the kinetic energy of the colliding nucleons.

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C.F. Powell and G.P.S. Occhialini in England in 1947 discovered a particle with a mass intermediate between that of an electron and a proton which has since been named a π-meson or a pion. It has been definitely established as the quantum of the internucleon force. Its rest mass is 273 m_e.

The e.m. field is described by a 4-vector, because the photons have polarization and hence must have non-zero spin $I = 1$ which can have three orientations in space. Actually because the transverse character of e.m. radiation they have only two possible polarizations.

The pions on the other hand have spin $I = 0$. Hence the meson field is a scalar or more probably a pseudoscalar (see later). We first assume it to be a scalar.

We write the relativistic equation for the total energy E of a particle of rest mass m.

$$E^2 = p^2 c^2 + m^2 c^4 \qquad \text{...(2.129)}$$

The quantum mechanical operators for E and p are

$$E \rightarrow i\hbar \frac{\partial}{\partial t}, \; p \rightarrow -i\hbar\nabla$$

Substitution in Eq. (2.8) gives

$$-\hbar^2 \frac{\partial^2}{\partial t^2} = -\hbar^2 c^2 \nabla^2 + m^2 c^4$$

or,
$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2 c^2}{\hbar^2} = 0 \qquad \text{...(2.130)}$$

Using the wave function $\phi(r)$, we then get introducing a source term $\eta(r)$ on the r.h.s.

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \beta^2\phi = 4\pi\eta(r) \qquad \text{...(2.131)}$$

where we have written
$$\beta = \frac{mc}{\hbar} \qquad \text{...(2.132)}$$

$\eta(r)$ is the source strength, analogous to the charge density for the e.m. field.

β has the dimensions of the reciprocal of length and is related to the range b of the internucleon interaction (see Eq. 2.128). Putting $m = 273m_e$. we get

$$b = 1/\beta = 1.41 \text{ fm}$$

Equation (2.131) can be written in a relativistically covariant form as in the case of e.m. wave equations. The static equation is

$$\nabla^2\phi - \beta^2\phi = 4\pi\eta(r) \qquad \text{...(2.133)}$$

Its solution is
$$\phi(r) = -\int \eta(r') \frac{\exp(-\beta|r - r'|)}{|r - r'|} \qquad \text{...(2.134)}$$

For a point source of strength q at $r' = r_1$ we can write $\eta(r') = g\delta(r' - r_1)$ which gives

$$\phi(r) = -g\frac{\exp\left(-\beta|r - r_1|\right)}{|r - r_1|} \qquad \text{...(2.135)}$$

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The potential energy of another point nucleon of strength g at $r = r_2$ is then given by (see Eq. 2.127)

$$V = g\phi(r_2) = -g^2 \frac{\exp(-\beta|r_2 - r_1|)}{|r_2 - r_1|} \qquad ...(2.136)$$

Comparing with Eq. (2.3)

$$V = -\frac{V_0}{(r/b)} \exp(-r/b)$$

We then have $g^2 = V_0\, b$; g^2 can be estimated from a knowledge of the values of V_0 and b for the singlet p-p potential: $V_0 = 43$ MeV, $b = 1.18$ fm. We get $g^2 \sim 8 \times 10^{-27}$ and $g^2/\hbar c = 0.3$.

This should be compared with the value of the fine structure constant $\alpha \approx 1/137$ which is the coupling constant for the electromagnetic interaction.

In the perturbation method used in quantum electrodynamics the successive terms decreases in the ratio of α. Hence the higher order terms are usually neglected in calculations. However, the method is not so successful in the case of the meson field theory because of the relatively large value of the coupling constant $g^2/\hbar c$. One of the consequences of this large coupling constant is that the exchange of two or more mesons may have appreciable probability.

Exchange of pions to provide internucleon interaction between two nucleons 1 and 2 can be expressed as follows:

Charged Pion Exchange

$$n_1 \overset{\pi^-}{\leftrightarrow} p_2\, (n_1 \rightarrow p_1 + \pi^-;\ p_2 + \pi^- \rightarrow n_2) \qquad ...(2.137)$$

$$p_1 \overset{\pi^-}{\leftrightarrow} n_2\, (p_1 \rightarrow n_1 + \pi^+;\ n_2 + \pi^+ \rightarrow p_2) \qquad ...(2.138)$$

In the first case the first nucleon which is a neutron (n_1) emits a π^- that is absorbed by the second nucleon (proton p_2). In the process n_1 becomes the proton p_1 while p_2 becomes the neutron n_2. The process then goes in the opposite direction. Similarly for the other case involving exchange of π^+.

Neutral Pion Exchange

$$n_1 \overset{\pi^0}{\leftrightarrow} n_2\, (n_1 \rightarrow n_1' + \pi^0;\ n_2 + \pi^0 \rightarrow n_2') \qquad ...(2.139)$$

$$p_1 \overset{\pi^0}{\leftrightarrow} p_2\, (p_1 \rightarrow p_1' + \pi^0;\ p_2 + \pi^0 \rightarrow p_2') \qquad ...(2.140)$$

To take into account exchange interaction, we have to introduce exchange operators which are constructed by using the isospin operators τ_1, τ_2, τ_3 identical with the Pauli spin operators σ_x, σ_y, σ_z. If $\gamma = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\delta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represent the two possible isospin states (the proton and the neutron respectively, we have

$$\tau_1\gamma = \delta,\ \tau_1\delta = \gamma$$

$$\tau_2\gamma = i\delta,\ \tau_2\delta = -i\gamma$$

$$\tau_3\gamma = \gamma,\ \tau_3\delta = -\delta \quad ...(2.141)$$

We then get the exchange operators τ_+ and τ_-:

$$\tau_+\gamma = \frac{1}{\sqrt{2}}(\tau_1 + i\tau_2)\gamma = 0,\ \tau_+\delta = \sqrt{2}\gamma \quad ...(2.142a)$$

$$\tau_-\gamma = \frac{1}{\sqrt{2}}(\tau_1 - i\tau_2)\gamma = \sqrt{2}\delta,\ \tau_-\delta = 0 \quad ...(2.142b)$$

Thus τ_+ transforms a neutron (δ) into a proton (γ) while τ_- transforms a proton (γ) into a neutron (δ). On the other hand τ_3 operating on the proton (neutron) wave function gives the same wave function, though with the sign changed in the case of the neutron (δ). In this respect, it is different from the "neutral field" of Yukawa's theory.

The source term in the wave function (2.131) has now to be written as $\eta(r)\,\tau_n$ where $\tau_n = \tau_\pm$ in 'charged theory' $\tau_n = \tau_\pm, \tau_3$ in 'symmetric theory' and $\tau_n = 1$ in 'neutral theory' of Yukawa. In this symmetric theory both $\pi^\pm$ and π^0 are exchanged.

The static wave equation is then

$$\nabla^2\phi - \beta^2\phi = 4\pi\tau_n\eta(r) \quad ...(2.143)$$

This gives the solution for a point nucleon at r_1

$$\phi(r) = -g_n\tau_n \frac{\exp\left(-\beta|r - r_1|\right)}{|r - r_1|} \quad ...(2.144)$$

The potential energy of a second nucleon of strength $g'_n\ \tau'_n$ at r_2 is thus

$$V = -g_n g'_n \tau_n \tau'_n \frac{\exp\left(-\beta|r_2 - r_1|\right)}{|r_2 - r_1|} \quad ...(2.145)$$

τ_n and τ'_n act on the wave functions of the emitting and absorbing nucleons respectively.

The meson field can either be scalar or pseudoscalar. A scalar field does not change sign on inversion while a pseudoscalar changes sign on inversion.

If $\tau^{(1)}$ and $\tau^{(2)}$ are the isospin operators for the two nucleons, then it can be easily seen that

$$\tau^{(1)}.\tau^{(2)} = \tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)} + \tau_3^{(1)}\tau_3^{(2)}$$

$$= \tau_+^{(1)}\tau_-^{(2)} + \tau_-^{(1)}\tau_+^{(2)} + \tau_3^{(1)}\tau_3^{(2)} \quad ...(2.146)$$

Then we get the following results for the different cases.

Charged Theory

$$V = -g^2(\tau_+^{(1)}\tau_-^{(2)} + \tau_-^{(1)}\tau_+^{(2)})\frac{\exp\left(-\beta|r_2 - r_1|\right)}{|r_2 - r_1|} \quad ...(2.147)$$

where it is assumed that $g_+ = g_- = g$. This gives exchange force between two unlike nucleons only through the exchange of charged mesons ($\pi^\pm$).

Symmetric Theory

$$V = -g^2 \tau^{(1)} . \tau^{(2)} \frac{\exp\left(-\beta|r_1 - r_2|\right)}{|r_1 - r_2|} \qquad ...(2.148)$$

NOTES

This gives exchange force between like nucleons (*p-p* and *n-n*) as also between unlike nucleons (*p-n*). The interaction energy V is a scalar both in ordinary space and isospin space in the symmetric theory. It maintains charge independence. This follows from the fact that $\tau^{(1)} . \tau^{(2)} = +1$ for the charge triplet state which can be *p-p*, *n-n* and *p-n* with $T_3 = +1, -1$ and 0 respectively in the spin singlet ($^1S_0$) state. For the charge singlet state $\tau^{(1)} . \tau^{(2)} = -3$ which is a spin triplet (deuteron ground state). Thus the interaction energy changes sign as we go from the charge triplet (spin singlet $^1S_0$) state to the charge singlet (spin triplet $^3S_1$) state. Since V becomes positive in the latter it is repulsive in the charge singlet state (deuteron ground state), which is obviously wrong.

Thus the symmetric theory does not give correct result if the meson field is assumed to be scalar.

Check Your Progress

5. How is the interaction between two charges described?
6. What is quantum of the nuclear field according to Yukawa?

2.9 ANSWERS TO 'CHECK YOUR PROGRESS'

1. Deuteron is the only two-nucleon bound system made up of a proton and a neutron.
2. In the scattering experiment, an incident beam of monoenergetic particles is scattered by an infinitely heavy scattering centre (in the C-system) at an angle θ.
3. Coulomb scattering calculations are made using parabolic coordinates.
4. The primary characteristics of the internucleon force can be inferred from a study of the ground state properties of the simplest bound state of the two nucleon system, which is the deuteron.
5. The interaction between two charges is described in terms of the emission of a photons by one charge and its absorption by the other.
6. According to Yukawa the quantum of the nuclear field is a particle (charged or uncharged) of finite rest mass to account for the short range character of the nuclear interaction.

2.10 SUMMARY

- Deuteron is the only two-nucleon bound system made up of a proton and a neutron. The two other possible two-nucleon systems, the diproton (2He) and the dineutron, do not exist as bound systems.

- Experimental evidences from proton-proton scattering shows that there is a repulsive core of the two-body potential at very short distances between the particles. Such a repulsive core potential may be either of finite height (soft core) or may be infinite (impenetrable or hard core).

NOTES

- The theoretical treatment of the neutron-proton scattering is based on the method of partial waves.
- In the scattering experiment, an incident beam of monoenergetic particles is scattered by an infinitely heavy scattering centre (in the *C*-system) at an angle θ.
- The stability of nuclei containing protons and neutrons shows that strong short range attractive nuclear force must be present between protons at close distance, just like the neutron-proton force. This force is in addition to the long range Coulomb repulsive force between the protons and actually dominates over the latter within the nucleus.
- For parallel spins of the neutron and the proton (each of spin 1/2) as in the deuteron, the resultant spin angular momentum is $S = 1$, which, has a statistical weight $2S + 1 = 3$. For antiparallel spin orientations, the resultant spin angular momentum of the *n-p* system is $S = 0$ which has a statistical weight $2S + 1 = 1$.
- For an attractive *n-p* potential giving rise to bound state of the deuteron ($E = E_{Bd} < 0$) the Fermi scattering length a must be positive. This happens because the internal wave functions (u_{in}) both for the deuteron and for low energy ($E < V_0$) scattering including $E = 0$ have negative slopes at the boundary $r = b$; *i.e.*, both $k_1 b$ and $k_0 b$ are slightly greater than $\pi/2$.
- From a knowledge of the sign of the scattering length it is possible to get an idea about the nature of a particular state of the *n-p* system, *i.e.*, whether it is bound or unbound.
- The experimental results on proton-proton scattering at high energies are very puzzling. For energies upto about 500 MeV, the angular distribution is flat except at small angles, showing that there is very little dependence of the differential cross-section on θ. Such spherically symmetric angular distribution is possible only for *S*-wave ($l = 0$) scattering which is clearly impossible at these high energies.
- The nearly isotropic angular distribution of *p-p* scattering can be explained on the basis of the following two hypotheses: (*a*) a force exists which makes the coefficients of the *S* and *D* interference terms negative; (*b*) there exists a tensor force in the triplet states which builds-up the angular distribution in the middle region.
- The primary characteristics of the internucleon force can be inferred from a study of the ground state properties of the simplest bound state of the two nucleon system, which is the deuteron.
- The electromagnetic interaction between two charged particles can be explained in terms of the interaction of these particles with the electromagnetic field.
- Moving charges produce a radiation field that can be described in terms of photons which are the quanta of the e.m. field.
- According to Yukawa the quantum of the nuclear field is a particle (charged or uncharged) of finite rest mass to account for the short range character of the nuclear interaction.
- The meson field can either be scalar or pseudoscalar. A scalar field does not change sign on inversion while a pseudoscalar changes sign on inversion.

2.11 KEY TERMS

- **Meson:** In particle physics, mesons are hadronic subatomic particles composed of an equal number of quarks and antiquarks, usually one of each, bound together by strong interactions.
- **Pion:** In particle physics, a pion is any of three subatomic particles: π^0, π^+, and π^-. Each pion consists of a quark and an antiquark and is therefore a meson. Pions are the lightest mesons and, more generally, the lightest hadrons.

2.12 SELF ASSESSMENT QUESTIONS AND EXERCISES

Short Answer Questions

1. Write in brief about the excited state of the deuterons.
2. State the effective range theory for bound case.
3. What is Mott scattering formula?
4. What do you mean by spin dependence?
5. What is scattering length?
6. Write briefly about p-p scattering at high energy.
7. How can the mechanism of internucleon interaction be represented?

Long Answer Questions

1. Describe the main experimentally determined properties of the deuteron.
2. Explain the low energy neutron-proton scattering.
3. Discuss the effective range theory.
4. Describe the p-p scattering at low energies.
5. Illustrate the wave equation for the deuteron along with its solution.
6. Explain Yukawa theory of nuclear forces.

2.13 FURTHER READING

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UNIT 3 NUCLEAR MODELS AND NUCLEAR REACTIONS

NOTES

Structure

3.0 Introduction
3.1 Objectives
3.2 Liquid Drop Model and Fission
3.3 Bohr and Wheeler's Theory
 3.3.1 Asymmetric Fission
 3.3.2 Spontaneous Fission
3.4 Nuclear Shell Model
3.5 Spin Orbit Coupling
3.6 Magnetic and Quadrupole Moments
 3.6.1 Quadrupole Moments
3.7 Nuclear Shell Structure
 3.7.1 Elementary Idea of Collective Model of the Nucleus
3.8 Conservation Laws of Nuclear Reactions and Q Value
3.9 Threshold Energy of a Nuclear Reaction
3.10 Nuclear Reaction
 3.10.1 Cross Section and Level Width
3.11 Bohr Compound Nucleus Theory of Nuclear Reaction
 3.11.1 Deuteron Stripping Reaction
3.12 Breit-Wigner Single Level Formula
3.13 Answers to 'Check Your Progress'
3.14 Summary
3.15 Key Terms
3.16 Self Assessment Questions and Exercises
3.17 Further Reading

3.0 INTRODUCTION

In nuclear physics, the Semi-Empirical Mass Formula (SEMF) (sometimes also called the Weizsäcker formula, Bethe–Weizsäcker formula, or Bethe–Weizsäcker mass formula to distinguish it from the Bethe–Weizsäcker process) is used to approximate the mass and various other properties of an atomic nucleus from its number of protons and neutrons. As the name suggests, it is based partly on theory and partly on empirical measurements. The formula represents the liquid drop model proposed by George Gamow, which can account for most of the terms in the formula and gives rough estimates for the values of the coefficients. It was first formulated in 1935 by German physicist Carl Friedrich von Weizsäcker and although refinements have been made to the coefficients over the years, the structure of the formula remains the same today.

The magnetic moment is the magnetic strength and orientation of a magnet or other object that produces a magnetic field. Examples of objects that have magnetic moments include: loops of electric current (such as electromagnets), permanent magnets, elementary particles (such as, electrons), various molecules, and many astronomical objects (such as, many planets, some moons, stars,.etc). The formula gives a good approximation for atomic masses and thereby other effects. However, it fails to explain the existence of lines of greater binding energy at certain numbers of protons and neutrons. These numbers, known as magic numbers, are the foundation of the nuclear shell model. A quadrupole or quadrapole is one of a sequence of configurations of things like electric charge or current, or gravitational mass that can exist in ideal form, but it is usually just part of a multipole expansion of a more complex structure reflecting various orders of complexity.

A stripping reaction is a nuclear reaction in which part of the incident nucleus combines with the target nucleus, and the remainder proceeds with most of its original momentum in almost its original direction. This reaction was first described by Stuart Thomas Butler in 1950. Deuteron stripping reactions have been extensively used to study nuclear reactions and structure, this occurs where the incident nucleus is a deuteron and only a proton emerges from the target nucleus.

In this unit, you will learn about the liquid drop model and fission, Bohr and Wheeler's theory, nuclear shell method, magnetic and quadrupole moments, nuclear shell structure, conservation laws of nuclear reactions and Q values, threshold energy of a nuclear reaction, nuclear reactions, cross section and level width, Bohr compound nucleus theory of nuclear reactions and Breit-Wigner single level formula.

3.1 OBJECTIVES

After going through this unit, you will be able to:

- Describe the liquid drop model and fission
- Explain the Bohr and Wheeler's theory
- State the nuclear shell method and structure.
- Elaborate on the magnetic and quadrupole moments
- Discuss the basic concept of conservation laws of nuclear reactions and Q values
- Analyze threshold energy of a nuclear reaction
- Discuss the Bohr compound nucleus theory of nuclear reactions
- Illustrate Breit-Wigner single level formula

3.2 LIQUID DROP MODEL AND FISSION

The macroscopic properties of the nucleus, *e.g.*, the constant density of the nuclear matter and the constant binding energy per nucleon are very similar to those found in a liquid drop. The very strong short range interaction between the nucleons permits us to consider their collective behaviour in determining the properties of the nucleus. As an example; if some extra energy is supplied to the nucleus, then instead of considering how the motions of the individual nucleons are affected by it, it is sufficient to consider its influence on the collective behaviour of the nucleons in the nucleus as a whole.

The liquid drop model was first proposed by N. Bohr and F. Kalckar in 1937 and was later applied by C.F. von Weizsäcker and H.A. Bethe to develop a semi-empirical formula for the binding energy of the nucleus.

There are reasons to believe that each individual molecule within a liquid drop exerts an attractive force upon a group of molecules in its immediate neighbourhood. The force of interaction does not extend to all the molecules within the drop. This is known as the *saturation* of the force. In order to calculate the potential of the interaction, it is necessary to know the number of interacting pairs of molecules within the drop. If each molecule interacts with all the molecules in the drop, the number of interacting pairs should be $N(N-1)/2$ where N is the total number of molecules. For N large, the number of pairs would thus be $N^2/2$ so that the potential energy should be proportional to N^2. On the other hand, if each molecule interacts with a limited number of molecules in its immediate vicinity, the number of interacting pairs would be linearly proportional to N so that the interaction potential should be proportional to N. This latter conclusion is supported by experimental evidence. The total amount of heat required for evaporating a drop of liquid (latent heat) is linearly proportional to the number of molecules within the liquid, as is evident from the fact that the heat required to evaporate 2 g of a liquid is twice that required to evaporate 1 g.

The binding energy E_B of a nucleus is proportional linearly to the number of nucleous within it, so that the binding fraction f_B (*i.e.*, binding energy per nucleon) is nearly constant (~8, MeV) for most nuclei. This fact shows a close resemblance of the nucleus with a liquid drop. Thus we come to the conclusion that the internucleon force within the nucleus attains a saturation value, so that each nucleon can interact only with a limited number of nucleons in its close vicinity. Apart from this, there are certain other points of resemblance between the nucleus of an atom and a liquid drop:

(*i*) The attractive force near the nuclear surface is similar to the force of surface tension on the surface of the liquid drop (see later);

(*ii*) As in the case of a liquid drop, the density of the nuclear matter is independent of its volume. The nuclear radius is $R \propto A^{1/3}$ where A is the mass number. Hence the nuclear volume $V \propto A$. Since the nuclear mass $M \sim A$, the density of the nuclear matter $\rho_m = M/V$ is independent of A. This also suggests saturation of the nuclear force;

(*iii*) Different types of particles, *e.g.*, neutrons, protons, deuterons, α-particles, etc., are emitted during nuclear reactions. These processes are analogous to the emission of the molecules from the liquid drop during evaporation;

(*iv*) The internal energy of the nucleus is analogous to the heat energy within the liquid drop;

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(*v*) The formation of a short lived compound nucleus by the absorption of a nuclear particle in a nucleus during a nuclear reaction is analogous to the process of condensation from the vapour to the liquid phase in the case of the liquid drop.

The liquid drop model is not very successful in describing the low lying excited states of the nucleus. Because of the collective motions of the large number of nucleons involved, the model gives rise to closely spaced energy levels. Actually however, these are found to be quite widely spaced at low excitation energies.

3.3 BOHR AND WHEELER'S THEORY

The drop's potential energy at each stage can be estimated as a function of its degree of deformation. The potential energy is displayed versus *r*, the distance between two fission fragments' centers. Three regions are meant to exist on the curve.

In area I, the fragments are entirely separated, and their potential energy E is just the electrostatic Coulomb energy produced by the two positively charged nuclear fragments' mutual repulsion. When the drops are in close proximity to one another and the distance $r=2R$, the energy E at the position is less than the corresponding Coulomb potential by an amount *CD*. This amount is equal to the potential for surface forces to become active at this stage. We approach the critical distance r_c, where the potential energy curve has a maximum value E_b, as we travel through region II. This relates to the barrier height and explains why spontaneous fission does not occur in all circumstances where $E_f > 0$. The nuclear system requires an additional amount of energy $E_a=E_b-E_f$ called the activation energy before the potential barrier can be overcome and fission can occur. The shards have collected in the III region, and short-range nuclear forces have taken precedence.

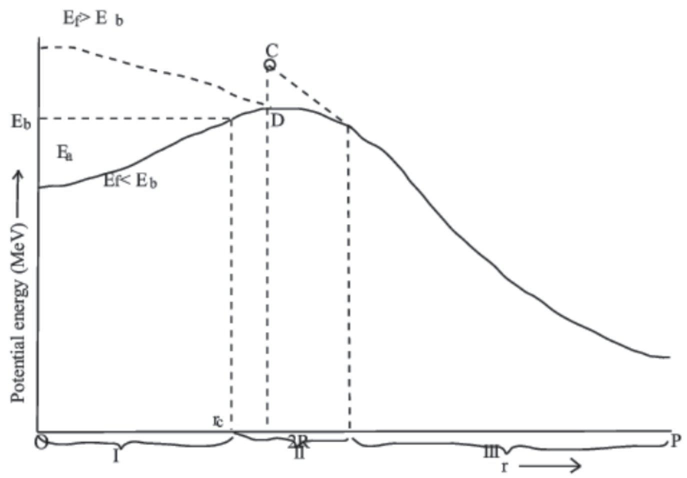

Fig. 3.1 *Potential energy curve of nuclear fission*

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Bohr and Wheeler provided the first theoretical treatment of this process. They used a straightforward analytical technique (Legendre polynomial expansion) to describe the radius r that forms an angle with the axis of maximal deformation.

$$r = R\left[1+\sum_{l=0}^{\infty}\alpha_l P_l(\cos\theta)\right] = R[1+\alpha_2 P_2(\cos\theta)+\alpha_3 P_3(\cos\theta)+...]$$

…..(3.1)

where R denotes the spherical nucleus's radius and α_2, α_3 denote the deformation parameters.

Here $\alpha_0 = \alpha_1 = 0$, in this case, as the drop's centre of mass is supposed to remain constant.

A spherical drop's surface energy $E_s = 4\pi R^2 T = 4\pi [R_0 A^{1/3}]^2 T$, where A is the mass number and T is the surface tension. As a result, in terms of deformation parameters, the surface energy of the deformed drop is given by

$$Es = 4\pi R_0^2 A^{2/3} T\left[1+\alpha_2\ (\frac{3}{2}.\cos^2\theta - \frac{1}{2} + ...\right].$$

$$Es = 4\pi R_0^2 A^{2/3} T\left[1+\frac{2}{5}\alpha_2^2+\frac{5}{7}\alpha_3^2+...\right].$$

The drop's surface energy changes as a result of deformation:

$$\Delta E_s = E_s\left[\frac{2}{5}\alpha_2^2+\frac{5}{7}\alpha_3^2+...\right].$$

…..(3.2)

The Coulomb energy of a spherical drop $E_c =$, hence that of the deformed drop

$$E_c = \frac{3}{5}\frac{Z^2e^2}{4\pi\varepsilon_0 R_0 A^{1/3}}\left[1+\alpha_2\left(\frac{3}{2}\cos^2\theta-\frac{1}{2}\right)+...\right]^{-1}$$

$$\Delta E_c = E_c\left[-\frac{1}{5}\alpha_2^2-\frac{10}{49}\alpha_3^2+...\right] = -E_c\left[\frac{1}{5}\alpha_2^2+\frac{10}{49}\alpha_3^2+...\right]$$

(3.3)

Keeping simply the term in mind, the entire difference in energy is as follows: $\Delta E = \Delta E_s + \Delta E_c = \frac{1}{5}\alpha_2^2[2E_s - E_c]$

The drop is stable to tiny distortions if it is positive, i.e., $2E_s > E_c$. If ΔE is negative or $E_s < E_c$, fissions may occur spontaneously.

$$4\pi R_0^2 A^{2/3} T < 3Z^2e^2/40\pi\varepsilon_0 A^{1/3} R_0 \ \text{ or } \ Z^2/A > 45$$

The critical parameter, denoted by χ, is the ratio . When $\chi < 1$, the nucleus is safe from spontaneous fission. From semi-empirical data $4\pi T = 13$MeV, it is possible to estimate the degree of distortion of a nucleus in the critical state by equating the critical or threshold energy E_{th} to the overall energy variation ΔE.

^{238}U; Es =520MeV and $E_c = 830 MeV$ thus $\alpha_2^2 = 1/7$.

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The energy that must be transferred to the nucleus in order to reach this crucial shape when the deformed drop is about to split into two equal drops is known as threshold energy or critical energy. The threshold energy is calculated as follows:

$$E_{th} = 4\pi R^2 T f(\chi) = 4\pi R_0^2 A^{2/3} T f(\chi) \qquad \ldots(3.4)$$

This energy can be computed by ignoring the second-order energy shift caused by the neck connecting the two fragments.

$$E_{th} = 2(4\pi R_0^2)T\left(\frac{1}{2}A\right)^{2/3} - 4\pi R_0^2 A^{2/3} T + 2\text{x}\frac{3}{5}x\left(\frac{1}{2}Ze\right)^2 / \ 4\pi\varepsilon_0 R_0\left(\frac{1}{2}A\right)^{1/3}$$

$$+\left(\frac{1}{2}Ze\right)^2 / 8\pi\varepsilon_0 R_0\left(\frac{1}{2}A\right)^{1/3} - \frac{3}{5}(Ze)^2 4\pi\varepsilon_0 R_0 A^{1/3}.$$

$$E_{th} / 4\pi R_0^2 T A^{2/3} = f(\chi) = 0.260 - 0.215\chi.$$

There are no electrostatic forces for an uncharged droplet =0 and $f(0)$ =0.260, therefore the critical energy is just the work done against surface tension in separating into two droplets. A minor deformation from the spherical shape leads the drop to reach the critical shape and separate at $\chi = 1$.

When the critical energy is compared to the excitation energy, the likelihood of fission can be predicted. The excitation energy E_e, which is contributed to the subsequent compound nucleus by a neutron's capture, is equal to the binding energy of the neutron in the compound nucleus and may be computed using the relationship.

$$Ee = B(A+1, Z) - B(A, Z) = {}_Z^A M + M_n - {}_Z^{A+1} M.$$

The calculated values of the excitation energy for a number of heavy nuclei are listed in the table and compared to the matching critical energy values. When looking at the results, it's clear that ^{238}U requires a critical deformation energy of 6.5 *MeV* for fission, but it only gets 5.9 *MeV* when it takes up a neutron with zero K.E. As a result, thermal neutrons with an energy of 0.03 *eV* are incapable of fission. Fission is feasible if the neutrons have a K.E. of 0.6 *MeV*. Experiments show that neutrons with an energy of roughly 1 *MeV* are required. With increasing neutron energy, the fission cross section grows rapidly. With ^{235}U, the situation is somewhat different. The excitation energy, or the energy available by capturing a slow neutron, is higher than the threshold energy in this case. Thermal neutrons should clearly be capable of generating ^{235}U nucleus fission in this instance.

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Table 3.1 *Excitation Energy and Critical energy for some Nuclides.*

Compound Nucleus	E_e(MeV)	E_{th}(MeV)	E_e-E_{th} (MeV)
^{232}Pa	5.4	5.0	0.4
^{233}Th	5.1	6.5	-1.4
^{235}U	6.6	5.5	1.1
^{238}Np	6.0	4.2	1.8
^{238}U	5.9	6.5	-0.6
^{240}Pu	6.4	4.0	2.4

3.3.1 Asymmetric Fission

Calculation of the fission barrier height $(E_b)_f$ as a function of the deformation parameter shows that the minimum value of E_b corresponds to symmetric fission and hence symmetric fission should be more probable than asymmetric fission, from the point of view of the liquid drop model. The reason why this is not so can be understood as follows. The initial deformation prior to fission is symmetric, as required by the liquid drop model. After the saddle point is attained, the disintegration into the fission fragments do not follow immediately, but takes place only after the nucleon shells in the would-be fragments have been formed. The numbers of nucleons in these shells happen fo be different and hence asymmetric fission occurs. The time scales involved in the two processes can be estimated as follows.

The fission time is $\tau_f = \delta/v$ where δ is the distance between the flying fragments ($\sim 10^{-13}$ m) and v their velocity ($\sim 10^7$ m/s) so that $\tau_f \sim 10^{-20}$ s. The shell formation time, on the other hand is $\tau_s \sim \hbar/\Delta E_s$, where $\Delta E_s \sim 1$ MeV is the mean separation between the one-particle levels. This gives $\tau_s \sim 10^{-34}/10^{-13} = 10^{-21}$ s $< \tau_f$.

3.3.2 Spontaneous Fission

The nuclear fission represents the process of breaking up of a nucleus into two fragment nuclei of comparable masses, whether or not induced by an external agent. Fission may also occur spontaneously. In the latter case we have Spontaneous Fission (*S.F.*) which was discovered by the Russian scientist G.N. Flerov. In spontaneous fission a nucleus ^A_ZX undergoes the spontaneous transformation

$$^A_Z\text{X} \rightarrow {}^{A_1}_{Z_1}\text{X}_1 + {}^{A_2}_{Z_2}\text{X}_2 \qquad ...(3.5)$$

where the two product nuclei have mass numbers and atomic numbers of comparable values. Here $A_1 + A_2 = A$ and $Z_1 + Z_2 = Z$. In the case when $A_1 = A_2 = A/2$ and $Z_1 = Z_2 = Z/2$ we have symmetric *S.F.* Here we have not included the prompt neutrons, which are not really the primary products of fission.

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The above processes can occur if the Q value of the transformation is positive *i.e.*,

$$Q_f = M(A, Z) - M(A_1, Z_1) - M(A_2, Z_2) > 0$$

where the M's are the atomic masses expressed in energy units. For the symmetric case, we have

$$Q_f = M(A, Z) - 2 \times M(A/2, Z/2) > 0$$

Written in terms of the binding energies (B) we have

$$Q_f = 2 \times B(A/2, Z/2) - B(A, Z)$$

$$= 2 \times (A/2)f'_B - Af_b = A(f'_B - f_{B^+}) = A \,.\, \Delta f_B$$

where f'_B s are the binding fractions.

For Q_f to be positive, Δf_B must be positive which happens if $f'_B > f_B$, *i.e.*, the binding fraction of the product nuclei is greater than that of the parent nucleus.

Writing the atomic masses in terms of the semi-empirical mass formula derived in this book, we have, neglecting the pairing energy term

$$M(A, Z) = ZM_H + NH_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A}$$

$$M\left(\frac{A}{2}, \frac{Z}{2}\right) = \frac{Z}{2} M_H + \frac{N}{2} M_n - a_1 \frac{A}{2} + a_2 \left(\frac{A}{2}\right)^{2/3} + a_3 \frac{(Z/2)^2}{(A/2)^{1/3}} + a_4 \frac{(A-2Z)^2}{2A}$$

Hence $\quad Q_f = M(A, Z) - 2M(A/2, Z/2)$

$$= a_2 A^{2/3}\left(1 - \frac{2}{2^{2/3}}\right) + \frac{a_3 Z^2}{A^{1/3}}\left(1 - \frac{2^{1/3}}{2}\right)$$

$$= -\,0.26\, a_2\, A^{2/3} + 0.37\, a_3\, Z^2/A^{1/3} \qquad \text{...(3.6)}$$

Thus the symmetric *S.F.* will be energetically possible ($Q_f > 0$) if

$$\frac{Z^2}{A} > \frac{0.26a_2}{0.37a_3}$$

Substituting the values $a_2 = 0.019114$ u and $a_3 = 0.0007626$ u, we get

$$\frac{Z^2}{A} > 17.6$$

This condition is found to be fulfilled for $A > 90$ and $Z > 40$. (For $A = 90$, $Z = 40$, $Z^2/A = 17.8$). Thus for nuclei for which $A > 90$, *S.F.* should be energetically possible. In reality however, it is a very uncommon phenomenon. Even amongst the nuclei of the heaviest atoms in the periodic table, *e.g.*, uranium, it is very rarely observed. For instance there is only about one *S.F.* per hour in 1 g of ^{235}U corresponding to a half-life of 2×10^{17} yr.

The reason for this lies in the quantum mechanical barrier penetration problem, which we discussed in connection with the α-disintegration of nuclei. The problem is much more acute in the present case, since the nuclei of the fission fragments carry much higher charges than the α-particles.

Let us consider the reverse case in which two spherical fission fragment nuclei of mass number $A/2$ and carrying positive charge $Ze/2$ each are brought towards each other from infinity. A and Z are the mass number and atomic number of the parent nucleus the symmetric spontaneous fission of which produces the above two. At infinity, their mutual potential energy is zero while at a distancer between their centres, the electrostatic potential energy rises to

$$V_c = \frac{1}{4\pi\varepsilon_0}\frac{(Ze/2)^2}{r} \qquad ...(3.7)$$

This potential energy is positive since the force is repulsive. As r decreases V_c increases as shown in Fig. 3.2(a). It should be maximum at B when the two fragments just touch each other which happens when their centres are separated by 2 R' where $R' = r_0 (A/2)^{1/3}$ is the radius of each fragment. However, actually the maximum is reached when the fragments begin to coalesce to produce the original nucleus under the action of strong short range attractive interaction (nuclear force). The highest point on the actual potential energy curve Q is below B. From this point where the potential is E_b, the course of the potential energy curve towards $r = 0$ is not known exactly.

The point P giving the energy at $r = 0$ corresponds to the mass energy of the parent nucleus. In order that the parent nucleus may undergo spontaneous fission, the two fragments must cross the potential barrier at the highest point Q.

The heights of the points P, P′, P″, etc., in Fig. 3.2(b) for different A values above the zero energy line (complete separation of the fission fragments) correspond to the Q_f values calculated above (Eq. 3.6). As long as these points are above the zero energy line, spontaneous fission is energetically possible. This is the case for nuclei with A > 90. However, because of the barrier penetration problem, there is very little probability of the fission to take place.

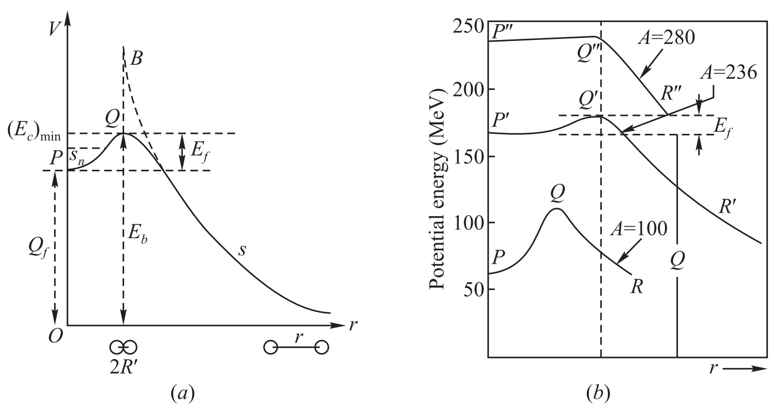

Fig. 3.2 *(a) Potential energy variation in nuclear fission. (b) Potential energy curves for different mass numbers.*

In Fig. 3.3, the barrier height E_b calculated from Bohr-Wheeler theory (a) and the values of Q_f (b) are plotted as functions of A for comparison. It will be seen that $E_b > Q_f$ for the nuclei with A upto ~ 250. $E_f = E_b - Q_f$ is called

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the activation energy for fission. So for these nuclei, the activation energy is positive. *S.F.* is possible in this case through barrier penetration only. This is the reason why *S.F.* is so rare a phenomenon. In the potential energy diagrams of Fig. 3.2(*b*) these correspond to the cases where the humps of the potential energy curves are above the rest energies of the parent nuclei.

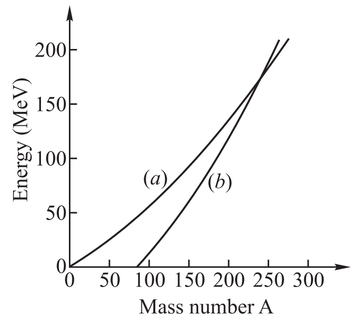

Fig. 3.3 *Comparison of the fission barrier (E_b) with the energy Q_f.*

Fission can be induced in these nuclei if energy is supplied to them by particle (neutron) or radiation (γ-ray) absoption. If the neutron separation energy from the target nucleus is S_n and the neutron kinetic energy is E_n, then the energy of excitation of the latter is

$$E_c = E_n + S_n \qquad ...(3.8)$$

For fission to occur, this must be greater than the deficiency of Q_f below E_b, *i.e.*,

$$E_c \geq E_b - Q_f \qquad ...(3.9)$$

The minimum energy required for the fission to occur is thus

$$(E_c)\ \text{min} = E_b - Q_f = E_f \qquad ...(3.10)$$

Bohr and Wheeler, on the basis of the liquid drop model of the nucleus, developed the theory for the calculation of the activation energy E_f.

3.4 NUCLEAR SHELL MODEL

It is thought that protons and neutrons in a nucleus are constantly colliding with each other. With such a strong force acting between them and so many nucleons to collide with, nucleons cannot conceivably complete entire orbits without interacting. No two electrons may occupy the same quantum state, according to Pauli's exclusion principle. The evidence for a shell structure and a limited number of permissible energy states suggests that a nucleon moves in some form of effective potential well created by all the other nucleons' forces. This leads to energy quantization in the same way that the square well Potential does. The designations for the levels differ slightly from the corresponding symbols for atomic energy levels. The energy levels grow as the orbital angular momentum quantum number *l* increases, and the *s, p, d, f...* symbols are used for *l =0,1,2,3...,* same as in the atomic case. However, because there is no physical equivalent to the primary quantum number *n*, the numbers associated with the level begin at *n=1* for the lowest level associated with a given orbital quantum number. In addition to the dependency on potential well details and orbital quantum number, there is a significant spin-orbit interaction that separates the levels by an amount that grows with orbital quantum number. This results in the overlapping layers depicted in the figure.

The subscript denotes the total angular momentum j, and the state has a multiplicity of $2j + 1$. Due to Coulomb repulsion, the contribution of a proton to energy is slightly different than that of a neutron, although this difference has no effect on the appearance of the set of energy levels. The nuclei with an even number of protons and neutrons are discovered to be more stable than those with an odd number. There are many 'Magic Numbers' of neutrons and protons that appear to be particularly favorable for nuclear stability: 2, 8, 20, 28, 50, 82, and 126. Nuclei whose neutron and proton counts are both equal to one of the magic numbers are referred to as 'Doubly Magical,' and are found to be exceptionally stable.

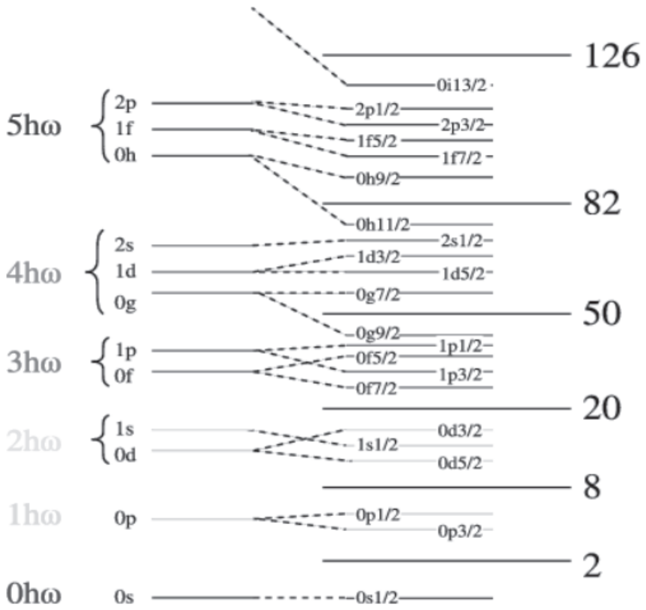

Fig. 3.4 *A schematic representation of the shell structure in nuclei.*

Predictions of the Shell Model:

1. **Stability of closed shell nuclei:** This system reproduces all the magic numbers, 2,8,20, 28,50,82,126, unambiguously.
2. **Spins and Parities of Nuclear Ground States:** The shell model has demonstrated remarkable success in forecasting the ground state spin of a large number of nuclei. The neutron and proton levels fill independently in this scenario. There are the following rules for angular momenta and ground state parities.

(i) Even-even nuclei have an angular momentum in their ground state of $J=0^+$. This rule has no known exceptions.

(ii) With an odd number of nucleons, such as an odd Z or odd N nucleus, the nucleons pair off as far as possible such that the resulting orbital angular momentum and spin direction are identical to those of the single odd particle.

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(iii) The total angular momentum of an odd-odd nucleus is equal to the vector sum of the odd neutron and odd proton j-values. The parity will be equal to the sum of the proton and neutron parities, i.e., parity = $(-1)^{ln + lp}$.

According to the first rule, the angular momentum of and is zero, as is the case for , , , , and all other even-even nuclei. We now provide some actual cases of odd even nuclei. Take the nucleus as an example. In the configuration , , the six protons and six of the seven neutrons are coupled. The odd neutron is designated . The ground state angular momentum is denoted by the subscript i.e., , a value that is experimentally measured. The unpaired particle in nucleus is a proton with spin . Consider the following examples: and . The shells are stuffed in accordance with

$$1s^{(2)}_{1/2}1p^{(4)}_{3/2}1p^{(2)}_{1/2}1d^{(1)}_{5/2}$$

If the nucleon is ^{17}O, the final unpaired nucleon is a neutron with a spin of 5/2; if the nucleon is ^{17}F, the final particle is a proton with a spin of 5/2. Thus, the model predicts 5/2, which is also the measured number for each of these nuclei's ground state spin.

3. Magnetic Moments of Nuclei: The overall angular momentum *J* of the nucleus is equal to the angular momentum *j* of the final unpaired nucleon in an odd nucleus. Thus, we observe that the odd nucleon alone generates the nucleus's magnetic moment. The orbital angular momentum (*l*) of numerical value and the spin *s* of numerical value combine to produce a total angular momentum *J* of numerical value , in units of ħ. $\mu_s = g_s s$ denotes the magnetic moment associated with spin angular momentum *s*.

Similarly, the magnetic moment associated with orbital angular momentum *l* is denoted by the equation $\mu_l = g_l\, l$.

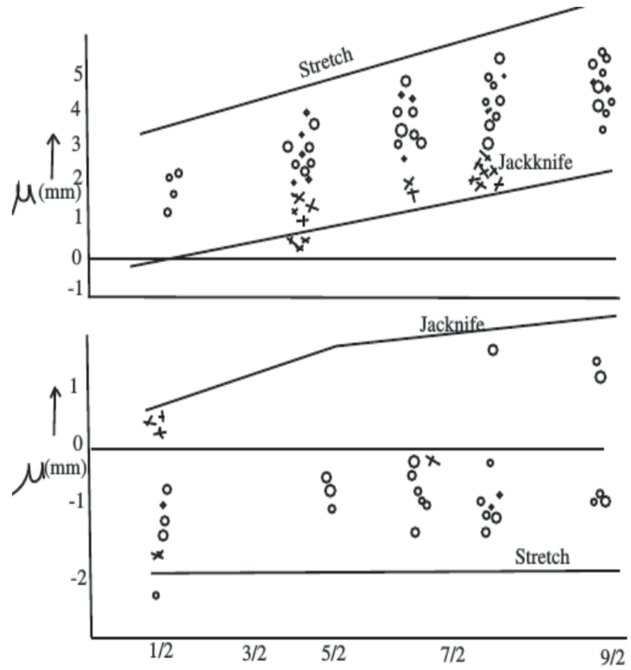

***Fig. 3.5** Schmidt lines plotting magnetic dipole moments against angular momentum (above) for odd Z-even N nuclei and (below) for even* Z-odd N nuclei.

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Hence μ = sum of the vectors $g_l l$ and $g_s s$ along j. The above relationship may be represented as by applying the cosine rule to the triangle created by the l, s, and j.

$$\mu = g_l\sqrt{[l(l+1)]}\frac{j(j+1)+l(l+1)-s(s+1)}{2\sqrt{[l(l+1)j(j+1)]}} + g_s\sqrt{[s(s+1)]}\frac{j(j+1)+s(s+1)-l(l+1)}{2\sqrt{[s(s+1)j(j+1)]}}$$

$$= \frac{j(j+1)+l(l+1)-s(s+1)}{2\sqrt{[(j(j+1)]}}g_l + \frac{j(j+1)+s(s+1)-l(l+1)}{2\sqrt{[j(j+1)]}}g_s$$

Since for a single particle, the spin s = ½ and there are two possible cases.

l parallel to s (Stretch case); J = $l + s = l +$ ½.

l antiparallel to s (Jacknife case); J = $l - s = l -$ ½.

Hence,

$$\mu = \left(J - \frac{1}{2}\right)g_l + \frac{1}{2}g_s \qquad \text{for stretch case}$$

….()

$$\mu = \frac{J}{J+1}\left[\left(J + \frac{3}{2}\right)g_l - \frac{1}{2}g_s\right] \qquad \text{for Jackknife case}$$

…..(3.12)

These relationships define two curves for μversus J with $J = l \pm$ ½, for each class of odd even nuclei. Schmidt values are the values of μand Schmidt lines are the curves. When the preceding equations (3.10) and (3.12) are substituted, the g factors corresponding to single nucleons are

g_l = l and g_s = 5.58 for protons and g_l = 0 and g_s = -3.82 for neutrons.

3.5 SPIN ORBIT COUPLING

In order to explain the disagreement at the higher magic numbers, Mayer and independently Haxel, Jensen and Suess suggested that a *spin-orbit interaction* term should be added to the central potential $V(r)$. The spin-orbit potential, which is non-central, can be written as

$$V_{ls} = -\phi(r)\, l.s$$

where $$\phi(r) = b\frac{1}{r}\left(\frac{\partial f}{\partial r}\right) \qquad ...(3.13)$$

Here $l\hbar$ and $s\hbar$ are the azimuthal and spin angular momenta of the nucleon under consideration. $f(r)$ is a spherically symmetric function giving the profile of the potential. It is weaker than $V(r)$ is a constant. We assume strong coupling between the spin and orbital angular momenta of each individual nucleon giving rise to a total angular momentum j for each so that we can write.

$$j = l + s \qquad ...(3.14)$$

Since s = 1/2 for each nucleon, the two possible values of j are

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$j = l + \frac{1}{2}$ and $l - \frac{1}{2}$. These two levels now have different energies because of the strong spin-orbit coupling. The splitting of the two levels can be calculated by computing the expectation values of the spin-orbit potential (3.13) in the two states of different j. It can be easily seen that

$$2l \,.\, s = j(j+1) - l(l+1) - s(s+l) \qquad ...(3.15)$$

which gives the following two values of $l \,.\, s$:

$$j = l + 1/2 : \quad l \,.\, s = l/2 \qquad ...(3.16)$$

$$j = l - 1/2 : \quad l \,.\, s = -(l+1)/2 \qquad ...(3.17)$$

The expectation values of the spin-orbit interaction potential is :

$$< V_{ls} > = \varepsilon_{ls} = -(l \,.\, s)\int_0^{\infty} \psi_{nl}^*(r)\,\varphi(r)\,\psi_{nl}(r)\,dr$$

$$= -(l \,.\, s) \langle \phi(r) \rangle \qquad ...(3.18)$$

where $\langle \phi(r) \rangle$ is the expectation value of $\phi(r)$ appearing in Eq. (3.13). We then have for the two states

$$j = l + 1/2 : \quad \varepsilon_{ls} = -l/2 \langle \phi(r) \rangle \qquad ...(3.19)$$

$$j = l - 1/2 : \quad \varepsilon_{ls} = \frac{l+1}{2}\langle \phi(r) \rangle \qquad ...(3.20)$$

The spin-orbit splitting of the two levels is then

$$\Delta\varepsilon_{ls} = \varepsilon_{ls}(l - 1/2) - \varepsilon_{ls}(l + 1/2)$$

$$= (l + 1/2) \langle \phi(r) \rangle \qquad ...(3.21)$$

The observed level-spacing is given by the following empirical formula:

$$\Delta\varepsilon_{ls} = 10(2l+1)\, A^{-2/3} \text{ MeV} \qquad ...(3.21a)$$

Since the r.h.s. of Eq. (3.21) is positive, it is obvious that the state with $j = l + \frac{1}{2}$ lies below the state $j = l - \frac{1}{2}$. The splitting which is of the order of a few MeV increases with increasing value of l. For the s-state ($l = 0$), only one value of j (= 1/2) is possible.

The spin-orbit potential assumed above resembles that which would arise due to a simple magnetic effect. However, the spin-crbit splitting is in this case much greater than the rather weak magnetic coupling between l and s. So it must be more intimately related to the central potential $V(r)$ giving rise to the shell structure. In analogy with the atomic case, it can be written as

$$\phi(r) = \frac{\beta}{r}\left(\frac{\partial V}{\partial r}\right) \qquad ...(3.22)$$

where β is the spin orbit constant.

There is evidence for the existence of a strong spin-orbit force between nucleons from high energy polarization experiments (see Ch. XVII), which justifies the above assumption.

The sequence of the energy levels, taking spin-orbit interaction into account, is shown in Fig. 3.6. Since we have to consider now the three quantum numbers n, l and j, the levels are designated as follows :

$1s_{1/2}$; $1p_{3/2}$, $1p_{1/2}$; $1d_{5/2}$, $1d_{3/2}$; ... $2s_{1/2}$; $2p_{3/2}$, $2p_{1/2}$; ... $2f_{7/2}$, $2f_{5/2}$; etc.

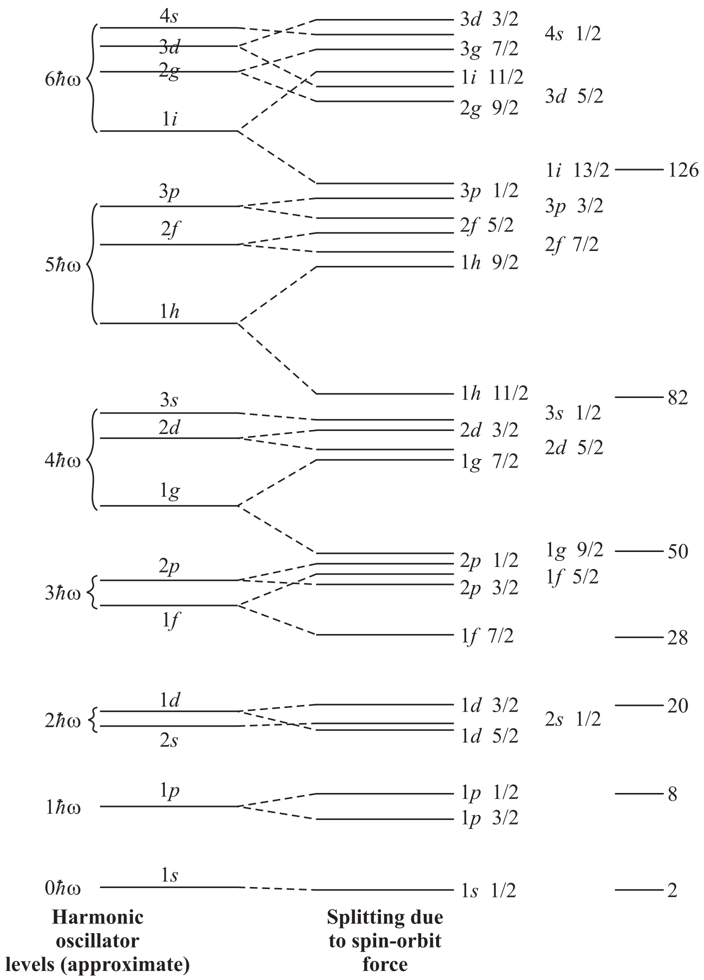

Fig. 3.6 *Sequence for nuclear levels according to shell model taking into account spin orbit interaction.*

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In accordance with Pauli's exclusion principle, each sublevel of a given j can accommodate a maximum of $(2j + 1)$ nucleons of either kind for which the magnetic quantum numbers m_j are different. The possible values are $m_j = j, j - 1, ... -j$. When a sublevel of given j is completely filled up with $(2j + 1)$ nucleons of a particular kind, the extra nucleons of the same kind must go to the next higher state of different j.

The group of sublevels (n, l, j) having energy values close to one another now constitute a shell. The number of nucleons required to fill up the shell is the sum of the nucleon numbers $(2j + 1)$. The total number of all the nucleons filling up the different sublevels upto a given shell from the lowest upwards constitute shell closure. These are shown in Fig. 3.6 on the extreme right which can be seen to agree with the observed magic numbers.

The lowest level, according to the new scheme is $1s_{1/2}$ with $j = 1/2$ which contains $(2 \times 1/2 + 1)$ or 2 nucleons. The next higher level with $\lambda = 1$ is now a combination of the two sublevels $1p_{3/2}$ and $1p_{1/2}$, the latter being above the former. The maximum number of nucleons which can occupy these sublevels are 4 and 2 respectively, so that the total number of nucleons in this group of sublevels is (4 + 2) or 6. So the shell closure takes place in this

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case with (2 + 6) or 8 nucleons as before.

The next group with $\lambda = 2$ are $2s$ and $1d$ which split up into the sublevels $2s_{1/2}$, $1d_{5/2}$ $1d_{3/2}$, the last one with $j = l - \frac{1}{2} (l = 2)$ lying above the $1d_{5/2}$ with $j = l + \frac{1}{2}$.

The sequence of the sublevels is shown in Fig. 3.6. The numbers of nucleons in the different sublevels are 2, 6 and 4 respectively, the total being 12. The shell closure takes place with (2 + 8 + 12) or 20 nucleons which is the same as before.

Departures from the simple theory without the spin-orbit coupling term being taken into account begin to appear from the next group of sublevels $2p$ and $1f$ with $\lambda = 3$. These now split up into $2p_{3/2}$, $2p_{1/2}$, $1f_{7/2}$ and $1f_{5/2}$. Because of the relatively larger value of l (= 3) for $1f$, the $1f_{7/2}$ sublevel $\left(j = l + \frac{1}{2} = 3 + \frac{1}{2} = \frac{7}{2} \right)$ is pushed down in energy considerably below the other three sublevels and lies as a separate level in between the two groups of $\lambda = 2$ and $\lambda = 3$ shells. This can be filled up completely with $\left(2 \times \frac{7}{2} + 1 \right)$ or 8 nucleons and a shell closure takes place at this point with (20 + 8) or 28 nucleons, as shown in Fig. 3.6.

The remaining sublevels $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ can contain a maximum of 12 nucleons which when added to the number 28 given above give rise to the semi-magic number 40 which is in agreement with observations.

To explain the magic number at 50 we have to consider the next group of levels $3s$, $2d$ and $1g$ with $\lambda = 4$. These split up into the sublevels $3s_{1/2}$, $2d_{5/2}$, $2d_{3/2}$, $1g_{9/2}$, $1g_{7/2}$. Because of the large value of 1 (= 4), the $1g_{9/2}$ sublevel is pushed way down to the vicinity of the previous group of sublevels ($2p_{3/2}$, $2p_{1/2}$, $1f_{5/2}$) and these four together constitute the shell which can contain a maximum of (12 + 10) = 22 nucleons, the number of nucleons required for filling up the $1g_{9/2}$ sublevel being 10. Thus the shell closure takes place at (28 + 22) or 50 nucleons in agreement with observed magic number.

Similarly because of the large spin-orbit splitting of the $1h$ level belonging to $\lambda = 5$, the sublevel $1h_{11/2}$ is pushed way down to the vicinity of the remaining sublevels at $\lambda = 4$. This group of five sublevels $3s_{1/2}$, $2d_{5/2}$, $2d_{3/2}$, $1g_{7/2}$ and $1h_{11/2}$ can then accommodate a maximum of (2 + 6 + 4 + 8 + 12) or 32 nucleons which when added to the 50 nucleons at the previous shell closure can account for a new shell closure at (50 + 32) or 82 nucleons in agreement with observations.

Finally the large spliting of the $1i$ level at $\lambda = 6$ into $1i_{13/2}$ and $1i_{11/2}$ pushes down the former to the vicinity of the $\lambda = 5$ group of remaining sublevels. The new group of six sublevels $1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$ and $1i_{13/2}$ can accommodate a maximum of (10 + 8 + 6 + 4 + 2 + 14) or 44 nucleons which when added to the number 82 accounts for occurence of the magic number 126 (Refer Fig. 3.6).

3.6 MAGNETIC AND QUADRUPOLE MOMENTS

NOTES

It was seen that the magnetic moment of a nucleus is the vector sum of the spin magnetic moment $\vec{\mu}_s$ and orbital magnetic moment $\vec{\mu}_L$:

$$\vec{\mu} = \vec{\mu}_s + \vec{\mu}_L \qquad ...(3.23)$$

$\vec{\mu}_s$ is the vector sum of the intrinsic magnetic moments of the individual nucleons in the nucleus. For protons and neutrons, the intrinsic moments are

$$\mu_p = g_p\, \mu_{N/2} \text{ and } \mu_n = g_n\, \mu_{N/2} \qquad ...(3.24)$$

where $\mu_N = e\hbar/2\, M_p$ is the nuclear magneton, M_p being the proton mass, g_p and g_n are the gyromagnetic ratios for the proton and the neutron respectively and have the numerical values

$$g_p = 2 \times 2.7927 \text{ and } g_n = -2 \times 1.9131 \qquad ...(3.25)$$

The magnetic moment of an odd *A* nucleus can be calculated using the extreme single particle shell model. The magnetic moment of a nucleus of spin *I* (total angular momentum) can be written as

$$\mu_I = g_I \sqrt{I(I+1)}\, \mu_N \qquad ...(3.26)$$

While measuring the magnetic moments, a magnetic field is applied and it is the component of μ_I in the field direction *z* which is determined. Using the rule of space quantization, this becomes

$$\mu_z = \mu_I \cos(I\,.\,B) = \frac{\mu_I m_I}{\sqrt{I(I+1)}}$$

where m_I is the magnetic quantum number which can take up the values $m_I = I, I - 1 \ldots -I$. *B* is the magnetic induction field. The largest component corresponding to $m_I = I$ usually gives the measured magnetic moment.

$$\mu_z = \frac{\mu_I\, I}{\sqrt{I(I+1)}} = g_I\, I\, \mu_N \qquad ...(3.27)$$

We have seen above that in extreme single particle model, an even number of nucleons of anyone kind always gives the resultant spin ($I = 0$). Hence the magnetic moment of an even-even nucleus will be 0:

$$(\mu_I)_{ee} = 0$$

Thus in an odd *A* nucleus, it is the last odd nucleon (proton or neutron) which determines the magnetic moment. For such a nucleus, $I = j$ where j is the total angular momentum of the last unpaired nucleon. Both the intrinsic magnetic moment (μ_s) and the magnetic moment due to its orbital motion (μ_l) have to be added up vectorially to get the total magnetic moment:

$$\vec{\mu}_j = \vec{\mu}_l + \vec{\mu}_s$$

where $\mu_s = \mu_p$ for the proton and $\mu_s = \mu_n$ for the neutron.

The orbital motion of a nucleon having azimuthal angular momentum $l\hbar$ produces a magnetic moment

$$\mu_l = g_l\, \mu_N \sqrt{l(l+1)} \qquad ...(3.28)$$

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Since the neutron is an uncharged particle its orbital motion does not produce any magnetic moment ($g_l = 0$) so that

$$(\mu_l)_n = 0 \quad ...(3.29)$$

In the case of the proton we can write $g_l = 1$ so that the orbital contribution is

$$(\mu_{lp}) = \mu_N \sqrt{l(l+1)} \quad ...(3.30)$$

Since the nucleons are spin 1/2 particles we can write the quantum mechanical values of the intrinsic magnetic moment as

$$\mu_s = g_s \mu_N \sqrt{s(s+1)} \quad ...(3.31)$$

where $g_s = g_p$ for the proton and $g_s = g_n$ for the neutron. $s = 1/2$ is the spin quantum number.

The total magnetic moment component in the direction of j is

$$\mu_I = \mu_j = \mu_l \cos(l, j) + \mu_s \cos(s, j)$$
$$= \mu_N \left\{ g_l \sqrt{l(l+1)} \cos(l, j) + g_s \sqrt{s(s+1)} \cos(s, j) \right\} \quad ...(3.32)$$

Using Eq. (3.26) we can also write

$$\mu_j = g_j \sqrt{j(j+1)}\, \mu_N \quad ...(3.33)$$

From the cosine law we have

$$\cos(l, j) = \frac{j(j+1) + l(l+1) - s(s+1)}{2\sqrt{j(j+1)}\sqrt{l(l+1)}}$$

$$\cos(s, j) = \frac{j(j+1) + s(s+1) - l(l+1)}{2\sqrt{j(j+1)}\sqrt{s(s+1)}}$$

We then have

$$\mu_j = g_l \mu_N \frac{j(j+1) + l(l+1) - s(s+1)}{2\sqrt{j(j+1)}} + g_s \mu_N \frac{j(j+1) + s(s+1) - l(l+1)}{2\sqrt{j(j+1)}} \quad ...(3.34)$$

For a spin 1/2 particle, j can have two values, $j = l \pm \frac{1}{2}$. So for a given j, l can have the following two values:

For $j = l + 1/2$, $l = j - 1/2$; for $j = l - 1/2$, $l = j + 1/2$.

For these two cases we get two different values of μ_j from Eq. (3.34).

Using Eq. (3.33) we get

$$g_j = g_l \frac{j(j+1) + l(l+1) - s(s+1)}{2j(j+1)} + g_s \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \quad ...(3.35)$$

As stated before, the measured magnetic moment μ_z is the largest possible component of μ_j in the magnetic field direction and is given by Eq. (3.27). Replacing I by j we then get

$$\mu_z = g_j j \mu_N$$
$$= \left\{ g_l \frac{j(j+1) + l(l+1) - s(s+1)}{2j(j+1)} \right.$$

$$+ g_s \frac{j(j+1) + s(s+1) - l(l+1)}{2(j+1)} \Bigg\} \mu_N \qquad ...(3.36)$$

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In the two cases stated above, we then get

For

$$l = j - \frac{1}{2}, \quad \mu_z = \left(g_l \frac{j - 1/2}{j} + \frac{g_s}{2j} \right) j\mu_N \qquad ...(3.37)$$

$$l = l + \frac{1}{2}, \quad \mu_z = \left(g_l \frac{j - 3/2}{j+1} - \frac{g_s}{2(j+1)} \right) j\mu_N \qquad ...(3.38)$$

In the case of odd A, either the proton number is odd (in the *o–e* nucleus) or the neutron number is odd (in the *e–o* nucleus). So we have the following possibilities.

Odd proton $(g_l = 1, g_s = g_p)$:

$$l = j - \frac{1}{2}: \quad \mu_z = \left(j - \frac{1}{2} - \frac{g_p}{2} \right) \mu_N \qquad ...(3.39a)$$

$$l = j + \frac{1}{2}: \quad \mu_z = \frac{j}{j+1} \left(j + \frac{3}{2} - \frac{g_p}{2} \right) \mu_N \qquad ...(3.39b)$$

Odd neutron $(g_l = 0, g_s = g_n)$:

$$l = j - \frac{1}{2}: \quad \mu_z = \frac{g_n \mu_N}{2} \qquad ...(3.40a)$$

$$l = j + \frac{1}{2}: \quad \mu_z = -\frac{j}{j+1} \frac{g_n \mu_N}{2} \qquad ...(3.40b)$$

The numerical values of g_p and g_n are given in Eq. (3.25). Eqs. (3.39) and (3.40) give the magnetic moments of odd A nuclei as functions of the nuclear spin I which is taken to be equal to the j value of the last odd nucleon. The above values of the nuclear magnetic moments are known as *Schmidt values*. In Fig. 3.7 these Schmidt values are plotted as functions of $I = j$ for the four cases given above. The graphs are known as *Schmidt diagrams*. Figure 3.7(*a*) shows the Schmidt plots for the *odd proton* case for $j = l \pm 1/2$ giving the two lines as shown. In the same diagram, the experimental values of the magnetic moments for some nuclei are also shown. Similarly, Fig. 3.7(*b*) shows the two Schmidt lines for the odd neutron case for $j = l \pm 1/2$. The experimental values are also shown.

The experimental values do not in general agree with the Schmidt values. However, almost invariably, the experimental values lie between the two limiting Schmidt lines, both for odd Z and odd N nuclei. The few exceptions are ^{3}H, ^{3}He, ^{13}C and ^{15}N for which the experimental values fall slightly above or below the limiting lines. For these and a few other nuclei, the experimental values lie close to one or the other Schmidt line. In all these nuclei, the last odd nucleons are in a $p_{1/2}$ level. Most of the experimentally measured magnetic moments lie nearer to one of the two Schmidt lines than the other which has the 1 value expected from the extreme single particle shell model.

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This fact enables us to confirm the nuclear ground state predicted by shell model in these cases. For example ^{7}Li with $Z = 3$ and $N = 4$ has a measured spin $I = 3/2$. The shell model prediction for the odd proton case gives $\mu_z = 3.7927$ for $l = j - \frac{1}{2} = 1$ which is a $p_{3/2}$ state while $\mu_z = 0.1244\ \mu_N$ for $l = j + \frac{1}{2} = 2$ which is a $d_{3/2}$ state. The experimental value is $\mu_z = 3.26\ \mu_N$ which confirms the assignment of $1p_{3/2}$ for the ground state of this nucleus. When the experimental values lie midway between the two limiting Schmidt lines no such unambiguous assignment is possible.

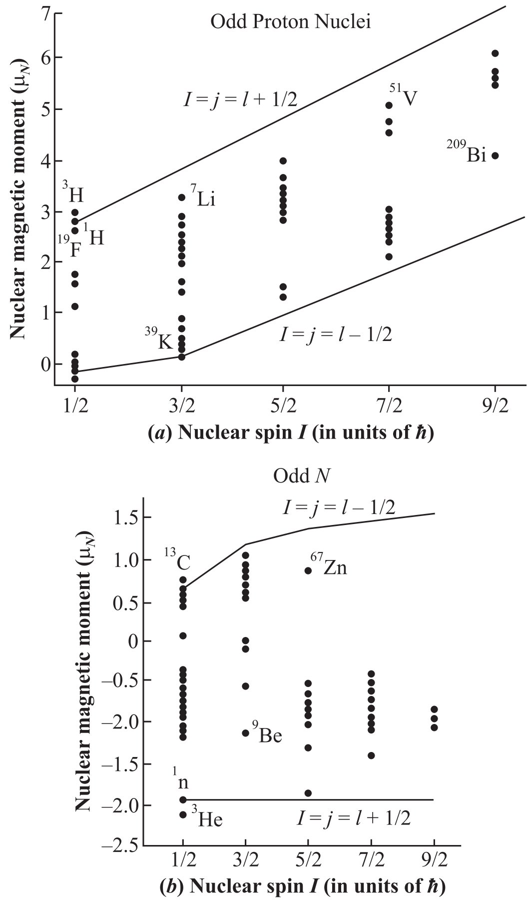

Fig. 3.7 Schmidt lines (a) odd proton case, (b) odd netron case.

It is also observed that when a shell is crossed, there is the expected sharp change in the magnetic moment from the value corresponding to $j = l - 1/2$ to that corresponding to $j = l + 1/2$. Some examples are given below:

Nuclide	*I*	*Shell model state*	μ_{obs}	μ_{calc}
$^{39}_{19}$K	3/2	$1d_{3/2}$ $(j = l - 1/2)$	0.22	0.12
$^{45}_{21}$Si	7/2	$1f_{7/2}$ $(j = l + 1/2)$	4.76	5.79

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The magnetic moments are in nuclear magneton units.

The departures from the Schmidt values are probably due to the (*i*) error in the expression for μ_z given above; (*ii*) error in the extreme single particle wave function used in the calculations. Rough calculations have been made with corrections due to (*a*) meson exchange currents, (*b*) departure of μ_p and μ_n values from their free state values and (*c*) modification of the wave functions due to the presence of the spin-orbit potential.

It may be noted that in the quantum mechanical theory, it is the expectation value $\langle \mu_z \rangle_{j,j}$ of the quantum mechanical operator $\hat{\mu}_z$ of the magnetic moment in the state j and $m_j = j$ which is expected value of the magnetic moment. The result is the same as given above, using the vector model approach.

3.6.1 Quadrupole Moments

The electric quadrupole moment Q of a nucleus is the average of the quantity $(3z^2 - r^2)$ for the charge distribution in the nucleus. For a spherically symmetric charge distribution this average is 0 and hence $Q = 0$ for even-even nuclei which have ground state spin $I = 0$. In the case of a single odd proton nucleus in the state j this averaging gives the quadrupole moment as

$$Q_{sp} = -\frac{2j-1}{2j+2}\langle r^2 \rangle \quad ...(3.41)$$

where $\langle r^2 \rangle$ is the mean square radius of the charge distribution which in the present case is equal to the mean square distance of the proton from the nuclear centre.

The negative sign on the r.h.s of Eq. (3.41) shows that orbital motion of the proton in the equatorial plane makes the charge distribution an oblate spheroid. On the other hand an odd hole in the state j would make the charge distribution a prolate spheroid for which $Q > 0$. Thus both positive and negative values of Q are expected.

In the case of a single odd neutron nucleus, one would not normally expect any quadrupole moment. However, the orbital motion of the neutron gives rise to the recoil motion of the rest of the nucleus which may be taken to be a charge Z at a distance r_n/A (r_n = radius of the odd neutron orbit) from the centre of mass. Hence a small quadrupole moment Q_{sn} may be expected, given by

$$Q_{sn} = \frac{Z}{A^2} Q_{sp} \quad ...(3.42)$$

This is much smaller than Q_{sp}.

The value of $\langle r^2 \rangle$ should be somewhat smaller than the square of the nuclear radius R^2. So Q_{sp} should be of the order of 10^{-28} to 10^{-29} m^2 and should increase with A, in proportion of $A^{2/3}$. For single neutron nuclei, Q_{sn} should be about one hundredth of the above value or less and should decrease roughly as $A^{-1/3}$.

The measured values of Q for odd A nuclei are in many cases much higher than the estimates given above. Further when Q is large, Q_{sp} and Q_{sn} are of the same order of magnitude.

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The above facts indicate that the single particle shell model cannot explain the very large values of Q in many nuclei. These nuclei seem to acquire a permanent deformation.

Check Your Progress

1. When was liquid drop model proposed?
2. What do you mean by the saturation of the force?
3. What are demerits of liquid drop model?
4. State Bohr and Wheeler's theory of nuclear fission.
5. What is spin orbit coupling?

3.7 NUCLEAR SHELL STRUCTURE

The different nuclear models which have been proposed from time to time can explain some limited features of the nucleus. Thus the liquid drop model can explain the observed variation of the nuclear binding energy with the mass number and the fission of the heavy nuclei. However, this model predicts very closely spaced energy levels in nuclei which is contrary to observation at low energies. The low lying excited states in nuclei are actually quite widely spaced, which cannot be explained by the liquid drop model. This and certain other properties of the nucleus would require us to consider the motion of the individual nucleons in a potential well which would give rise to the existence of a nuclear shell structure, similar to the electronic shells in the atoms.

We know that the extranuclear electrons in the atoms are arranged in a number of shells *e.g.*, K, L, M, N etc. with the respective principal quantum number $n = 1, 2, 3, 4$ etc.

Each of these shells has a number of subshells characterized by different values of the azimuthal quantum number $l = 0, 1, 2, 3, ... (n - 1)$. A subshell of given l can contain a maximum of $2(2l + 1)$ electrons, which means that the s, p, d, f etc. subshells with $l = 0, 1, 2, 3$ etc. can accommodate upto 2, 6, 10,14 etc. electrons respectively.

In the inert gases Ne ($Z = 10$), Ar ($Z = 18$), Kr ($Z = 36$), Xe ($Z = 54$) and Rn ($Z = 86$), the outremost p subshells are completely filled up while in the lightest inert gas He ($Z = 2$), the 1s subshell is filled up with 2 electrons. In all these elements, the electrons are very tightly bound, their first ionization potentials being relatively quite high.

In the alkali elements, which follow immediately the inert gases in the periodic table, there is *one* electron in s subshell just outside the inert gas core. This electron is very weakly bound in all of these elements [Refer Fig. 3.8(*a*)]. The sudden drop in the first ionization potentials after the inert gases is evident from the figure.

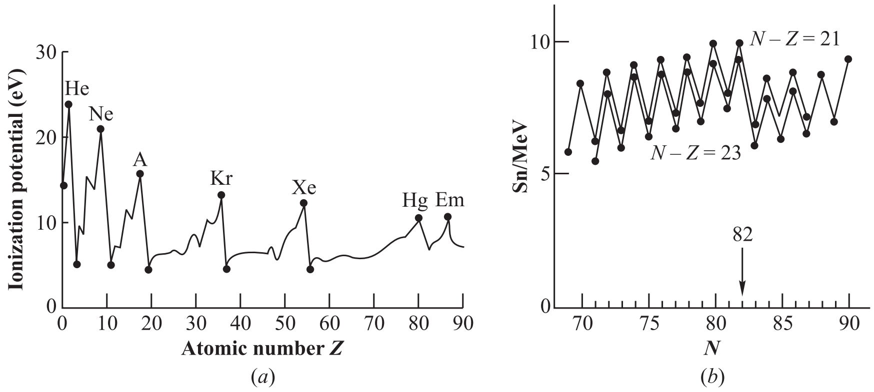

Fig. 3.8 *(a) First ionization potentials of the atoms in the periodic table. Notice the discontinuities at shell closures. (b) Discontinuity in neutron separation energy at N = 82.*

There are strong reasons to believe that as in the case of binding of the electrons in the atoms, the nucleons in the nuclei are arranged in certain discrete shells.

W.M. Elasser, in 1933, was the first to point this out. Later, Maria Gopert Meyer (1948) and independently O. Haxel, J.H.D. Jensen and H.E. Suess (1949) showed that the nuclei containing the following numbers of protons and neutrons exhibited very high stability:

Protons	2	8	20	28	50	82	
Neutrons	2	8	20	28	50	82	126

The above numbers are popularly known as *magic numbers* and are analogous to the atomic numbers of the inert gases. In addition to the above, there is a semi-magic number at N and $Z = 40$.

Some nuclei contain magic numbers of protons and neutrons both. Examples ^{4}He ($Z = 2, N = 2$), ^{16}O ($Z = 8, N = 8$), ^{40}Ca ($Z = 20, N = 20$), ^{48}Ca ($Z = 20, N = 28$), ^{208}Pb ($Z = 82, N = 126$). They are *doubly magic* and show exceptionally high stability.

Following are the maio evidences to show the existence of shell structure within the nuclei.

(*a*) Nuclei containing magic numbers of protons of neutrons show very high stability, compared to the nuclei containing one more nucleon of the same kind. Measurement shows that the separation energy S_n of a neutron from a nucleus containing a magic number of neutrons is large compared to that for a nucleus containing one more neutron. Similarly the separation energy S_p of a proton from a nucleus containing a magic number of protons is large compared to that for a nucleus containing one more proton. (By separation energy is meant the minimum energy

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needed for separating one neutron or proton from a nucleus).

The sudden discontinuity in the value of S_n at the magic neutron number is shown in Fig. 3.8(*b*).

(*b*) The naturally occurring isotopes, whose nuclei contain magic numbers of neutrons or protons, have generally greater relative abundances (> 60%). For example, the isotopes ^{88}Sr (N = 50), ^{138}Ba (N = 82) and ^{140}Ce (N = 82) have relative abundances of 82.56%, 71.66% and 88.48% respectively.

(*c*) The number of stable isotopes of an element containing a magic number of protons is usually large compared to those for other elements. For example, calcium with Z = 20 has 6 stable isotopes compared to 3 and 5 for argon (Z = 18) and titanium (Z = 22) respectively. Again tin with Z = 50 has the largest number of stable isotopes. This number is 10 compared to 8 for cadmium (Z = 48) and tellurium (Z = 52).

(*d*) The number of naturally occurring isotones with magic numbers of neutrons is usually large compared to those in the immediate neighbourhood. As an example, the number of stable isotones at N = 82, is 7 compared to 3 and 2 at N = 80 and N = 84 respectively. Similar is the situation at N = 20, 28 and 50 which have respectively 5, 5 and 6 isotones. These numbers are greater than in the cases of the neighbouring iotones.

(*e*) The stable end products of all the three natural radioactive series described in Ch. II are the three isotopes of lead (^{206}Pb, ^{207}Pb and ^{208}Pb) which all have the magic number Z = 82 of protons in their nuclei.

(*f*) Nuclei with magic numbers of neutrons or protons have their first excited states at higher energies than in the cases of the neighbouring nuclei.

(*g*) The neutron capture cross-sections of the nuclei with magic numbers of neutrons are usually low. Since the neutron shells are filled up in these nuclei, the probabilities of their capturing an additional neutron is small (Refer Fig. 3.9). Similarly nuclei with magic proton numbers have low proton capture cross-sections.

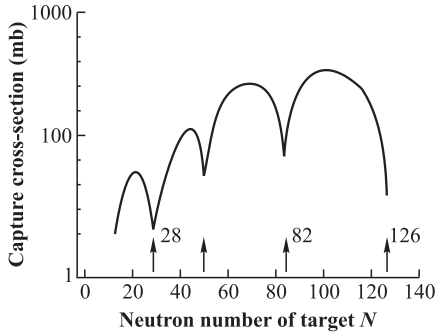

Fig. 3.9 *Variation of neutron capture cross section with N showing discontinuities at the magic numbers*

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(*h*) If the α-disintegration energies of the heavy nuclei are plotted as functions of the mass number *A* for a given *Z*, then usually a regular variation is observed till the magic neutron number $N = 126$ is reached when there is a sudden discontinuity (Refer Figure 3.10). This confirms the magic character of the neutron number 126.

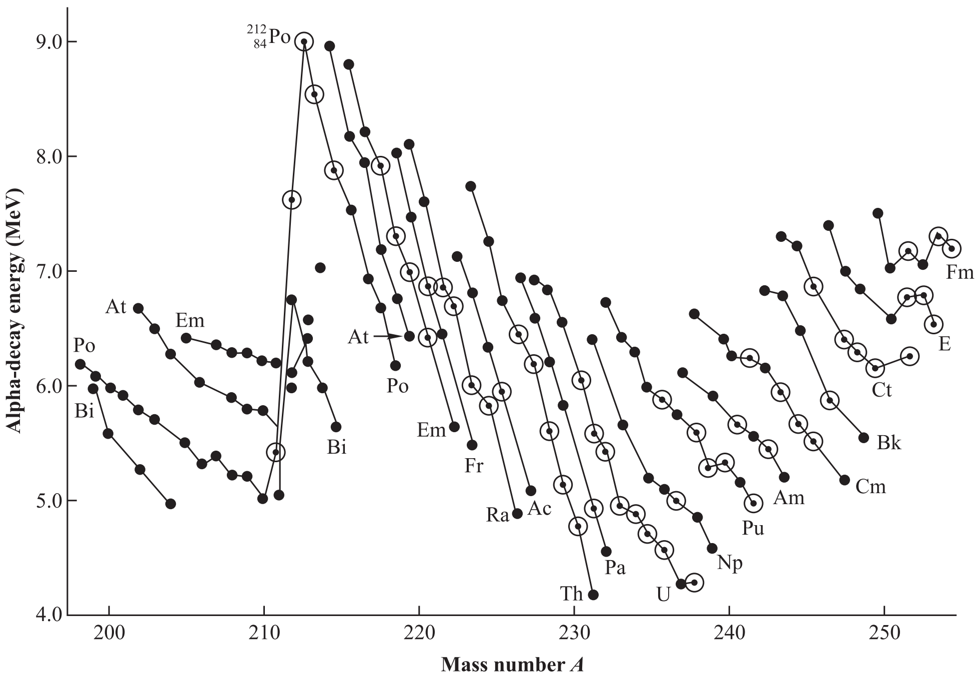

Fig. 3.10 *Discontinuities in the a-disintegration energies at N = 126 for heavy nuclides.*

(*i*) Similar discontinuities are observed amongst the β-emitters at the magic neutron or proton numbers.

The experimental results summarized above lend strong support to the proposition of shell structure for the nucleus.

To develop a theory of the nuclear shell structure, it is necessary to assume the existence of a potential well within the nucleus. It is known from quantum mechanics that a bound physical system in an attractive potential well can exist in a number of discrete quantum states. This is the case for the electrons in an atom which are acted upon by the Coulomb field of the nucleus. If the interactions between the electrons are neglected, then we can regard the field as spherically symmetric. Solving the Schrödinger equation with a potential giving rise to such a field it is possible to find the energy levels for different sets of quantum numbers which determine the electronic shells in the atoms.

3.7.1 Elementary Idea of Collective Model of the Nucleus

Both the single particle shell model and the individual particle shell model are based on the assumption of the existence of a spherically symmetric potential in the nucleus, plus a spin-orbit coupling term. The different types

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of coupling of the angular momenta assumed for the loose nucleons outside the core gives rise to the different forms of the shell model.

The shell model, with some refinements, has been successfully applied to explain many features of the nucleus in the ground state and in some of the excited states. However, it fails conspicuously in explaining the observed large electric quadrupole moments (Q) of the nuclei in many cases and the quadrupole transition rates B (E2). In such cases where Q is n times the single particle value (see Fig.), we must assume that $2n$ particles are involved in producing the observed Q since the neutrons cannot directly contribute to Q. It is the collective motion of a fairly large number of nucleons which determines the large values of Q for nuclei far from closed shells.

J. Rainwater (1950) was the first who tried to explain these failures of the shell model by introducing the idea of deformation in the shape of the nuclear core due to the motion of the loose odd nucleon outside the core in odd A nuclei. According to him such motion leads to a polarization of the even-even core, which thus assumes a spheroidal shape. Such deformation would cause the quadrupole moment to be higher than the single particle value. E2 transition rate is also increased. Aage Bohr (son of famous Niels Bohr) and B. Mottleson (1953) further elaborated the model, combining the single particle and collective motions into a *unified model* which gave a more complete description of the deformed nuclei.

In nearly spherical nuclei, the coupling between the collective motion of the nucleons in the core and the motion of the loose nucleons outside the core is weak. On the other hand, for strong coupling, the surface is distorted and the potential felt by the loose particles is not spherically symmetric. These particles, moving in a non-spherically symmetric shell model potential, maintains the deformed nuclear shape. The situation is similar to that in a linear molecule. We can then write the total energy as the sum of the rotational, vibrational and nucleonic energies of the nucleus, as in the case of the molecule. In the present case, the nucleonic energy replaces the electronic energy of the molecules:

$$E_{\text{tot}} = E_{\text{rot}} + E_{\text{vib}} + E_{\text{nuc}} \qquad \text{...(3.43)}$$

The collective motion of the nuclear core gives rise to the rotational and vibrational term, while nucleonic energy term is due to the motion of the loose nucleons.

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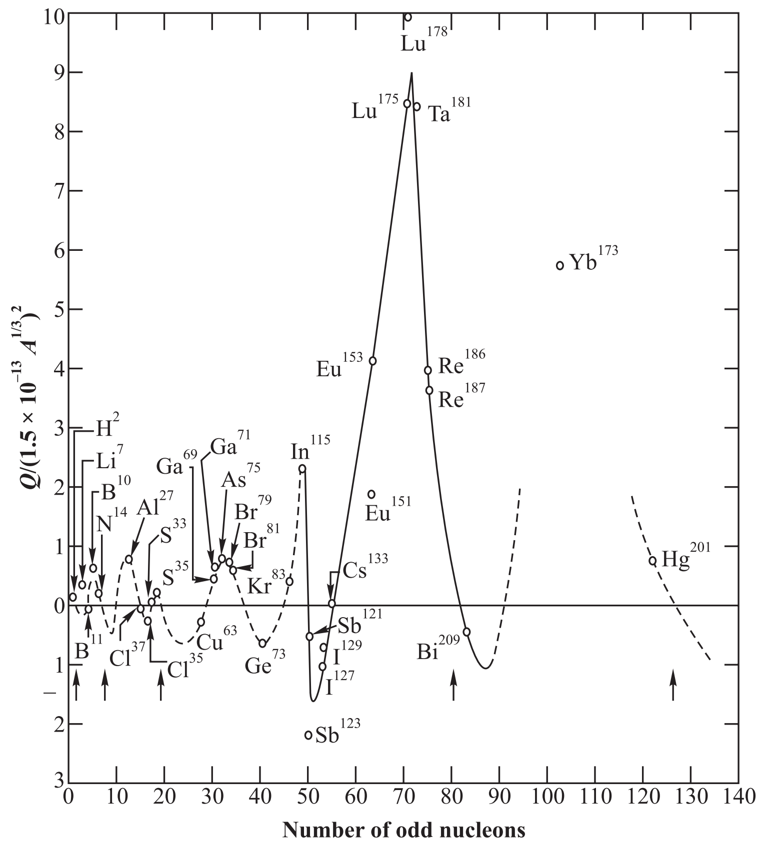

Fig.3.11 *Nuclear quadrupole moments of e–o nuclei. The arrows indicate the closed shells.*

The coupling of the external nucleonic motion and collective motion gives rise to *shape-oscillations* at the nuclear surface. The rotational motion is rather complicated in that it is not a rotation of the whole nucleus, considered as a rigid body. Rather, it is the rotation of the deformed portion of the nuclear surface. In other words, a rotation of the shape occurs with the deformation being maintained. The moment of inertia is lower for such rotation than in the case of rigid body rotation.

We consider below the vibrational and rotational motions of even-even nuclei. Experimental evidence shows that far from the dosed shells, the motion of the loose nucleons produces large permanent deformations, characterized by rotatiunal spectra. The nuclei are found in the middle of 1*d*, 2*s* shells in the range 145 < *A* < 185 and for *A* > 226. The energy, difference between the 0^+ ground state and the 2^+ first excited state is of the order of 100 keV in them. Far from the deformed regians and nearer the closed shells, the equilibrium shape is spherical. Low energy excitations produce characteristic vibrational spectra. At the closed shells, excited states can be producea by the break up of the core, giving rise to new particle states.

3.8 CONSERVATION LAWS OF NUCLEAR REACTIONS AND Q VALUE

NOTES

The occurrence of a nuclear reaction is usually governed by certain conservation laws.

(*a*) *Conservation of mass number*: The total number of neutrons and protons in the nuclei taking part in a nuclear reaction remains unchanged after the reaction. Thus in the reaction X (*x*, *y*) the sum of mass numbers of X and *x* must be equal to the sum of the mass numbers of Y and *y*:

$$A + a = A' + a' \quad ...(3.44)$$

In the general case of reactions involving elementary particles the law can be expressed by requiring the total number of heavy particles (baryons) remains unchanged in a reaction.

(*b*) *Conservation of atomic number*: The total number of protons of the nuclei taking part in a nuclear reaction remains unchanged after the reaction. This means that the sum of atomic numbers of X and *x* is equal to the sum of atomic numbers of Y and *y*:

$$Z + z = Z' + z' \quad ...(3.45)$$

In view of the conservation law (*a*) and (*b*) above it is easily seen that the mass number and the atomic number of the product nucleus in Rutherford's experiment should be $A' = A + a - a' = 14 + 4 - 1 = 17$ and $Z' = Z + z - z' = 7 + 2 - 1 = 8$, so that the product nucleus must be the isotope ^{17}O of oxygen.

Further, in view of (*a*) and (*b*) the neutron number *N* remains unchanged in the reaction.

(*c*) *Conservation of energy; Q value of a nuclear reaction*: In order to apply the law of conservation of energy in the case of a nuclear reaction, it is necessary to take into account the mass-energy equivalance predicted by the special theory of relativity. Conservation of energy requires that the total energy, including the rest–mass energies of all the nuclei taking part in a reaction and their kinetic energies, must be equal to the sum of the rest–mass energies and the kinetic energies of the products.

Writing M_X, $M_x M_Y$ and M_y as the rest–masses of the different atoms in Eq. given in this book, their rest mass energies are $M_X c^2$, $M_x c^2$, $M_Y c^2$ and $M_y c^2$ respectively, Denoting the kinetic energy by *E* we then get $M_X c^2 + M_x c^2 + E_X + E_x = M_Y c^2 + M_y c^2 + E_Y + E_y$.

During the nuclear reaction, the target nucleus is usually at rest, so that $E_X = 0$. The above equation then becomes

$$M_X c^2 + M_x c^2 + E_x = M_Y c^2 + M_y c^2 + E_Y + E_y \quad ...(3.46)$$

The above energy balance equation is often written without the factor c^2 in the mass–energy terms, which means that the masses are expressed in energy units.

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It may be noted that though the nuclear masses are involved in a nuclear reaction, it is possible to write the energy–balance equation in terms of the atomic masses, since the electronic masses cancel out on the two sides of the equation and the electronic binding energies can be neglected.

It may be noted that at relatively lower energies, the kinetic energy is given by the non-relativistic expresions: $E = Mv^2/2$. When the energies of the particles involved in the reaction are very high, as in the case of many elementary particle reaction, the relativistic expression for the kinetic energy must be used: $E = \sqrt{p^2c^2 + M_0^2c^4} - M_0c^2$. Here M_0 is the rest mass of the particle and $p = M_0v/\sqrt{1-\beta^2}$ is its linear momentum.

(*d*) *Conservation of linear momentum*: If p_X, p_x, p_Y and p_y represent the momentum vectors of the different nuclei taking part in a reaction, then the law of conservation of linear momentum gives

$$p_X + p_x = p_Y + p_y \qquad ...(3.47)$$

Equation (3.47) holds in an arbitary frame of reference. In the laboratory frame of reference (L-system) in which the target nucleus is at rest p_X = 0 and the above equation becomes

$$p_x = p_Y + p_y \qquad ...(3.48)$$

In the frame of reference in which the centre of mass of the two particles before collision is at rest (C-system), we have to write $p_X + p_x = 0$, which gives $p_Y + p_y = 0$ *i.e.*, the centre of mass of the product particles is also at rest in this system.

(*e*) *Conservation of angular momentum*: In a nuclear reaction of the type $X + x \rightarrow Y + y$, the total angular momentum of the nuclei taking part in the reaction remains the same before and after the reaction.

Let I_X, I_x, I_Y, I_y denote the nuclear spins (total angular momentum) of the nuclei X, *x*, Y and *y* respectively. Let l_X represent the relative orbital angular momentum of X and *x* (*i.e.*, in the initial state). Similarly l_Y denotes the relative orbital angular momentum of Y and *y* (*i.e.*, in the final state). Then according to the law of conservation of angular momentum, we must have.

$$I_X + I_x + l_X = I_Y + I_y + l_Y$$

Application of the law of conservation of the angular momentum taking into account the well-known quantum mechanical properties of the former leads to certain selection rules.

(*f*) *Conservation of parity*: Since the nuclear reactions take place due to the strong interaction in which parity is conserved, the parity Π_i before the reaction must be equal to the parity Π_f after the reaction.

Denoting the intrinsic parities of the nuclei taking part in the reaction by Π_X, Π_x, Π_Y and Π_y we get for the initial and final states of the reaction

$$\Pi_i = \Pi_X \Pi_x (-1)^{l_x}$$
$$\Pi_f = \Pi_Y \Pi_y (-1)^{l_y}$$

The conservation of parity requires that

$$\Pi_X \Pi_x (-1)^{l_x} = \Pi_Y \Pi_y (-1)^{l_y}$$

Except in the cases of elementary particle reactions, the intrinsic parity need not be taken into account. Hence we get

$$(-1)^{l_x} = (-1)^{l_y}$$

Parity conservation results in certain selection rules which limit the possible nuclear reactions that may occur starting from a given initial state *i*. For example, in the case of elastic scattering *l* can change only by an even integer.

(*g*) *Conservation of isotopic spin*: Denoting the isotopic spin vectors for the initial and final states by T_i and T_f, we have from the law of conservation of isotopic spin applicable in the case of strong interaction

$$T_i = T_f$$

Since for the reaction $X + x \rightarrow Y + y$, $T_i = T_X + T_x$ and $T_f = T_Y + T_y$,

we have $\quad T_X + T_x = T_Y + T_y$

Isotopic spin is a characteristic of the nuclear level. Hence the above conservation law can be used to identify the levels of the nuclei produced in the reaction. In particular if $T_x = T_y = 0$ (as for the deuteron or the α-particle), we must have $T_X = T_Y$.

This rule must be obeyed in reactions of the type (*d*, α) (*d*, *d*) (α, *d*), (α, α) etc. The rule has been verified for the nuclei ^{6}Li, ^{10}B and ^{14}N for $T = 0$ in the ground states.

3.9 THRESHOLD ENERGY OF A NUCLEAR REACTION

The *Q* value of a reaction can be expressed in terms of the kinetic energies of the projectile (E_x) and of ithe product nuclei E_y and E_Y.

In view of the energy and momentum conservation laws, E_Y can be expressed in terms of E_x and E_y. Refering to Fig. 3.12, we get from the law of conservation of momentum along and perpendicular to the direction of motion of the projectile $(\because\ p = \sqrt{2ME})$

$$\sqrt{2M_x E_x} = \sqrt{2M_y E_y}\cos\theta + \sqrt{2M_Y E_Y}\cos\varphi \qquad ...(3.49)$$

$$0 = \sqrt{2M_y E_y}\sin\theta - \sqrt{2M_Y E_Y}\sin\varphi \qquad ...(3.50)$$

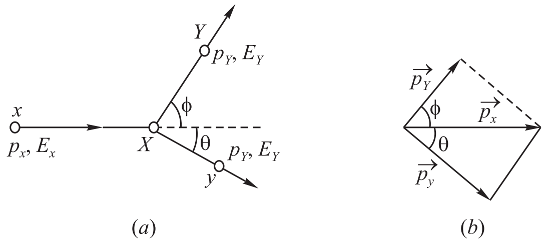

Fig. 3.12 *(a) Motion of the projectile (x) and product particles (y and Y) taking part in a nuclear reaction. (b) Momentum diagram.*

The following equation gives the law of conservation of energy:

$$Q = E_Y + E_y - E_x$$

Squaring and adding Eqs. (3.49) and (3.50), we get

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$$2M_Y E_Y = 2M_x E_x + 2M_y E_y - 4\sqrt{M_x M_y E_x E_y}\cos\theta$$

or

$$E_Y = \frac{M_x}{M_Y}E_x + \frac{M_y}{M_Y}E_y - \frac{2}{M_Y}\sqrt{M_x M_y E_x E_y}\cos\theta \qquad ...(3.51)$$

Then from above Eq. and (3.51) we get

$$Q = E_y\left(1+\frac{M_y}{M_Y}\right) - E_x\left(1-\frac{M_x}{M_Y}\right) - \frac{2}{M_Y}\sqrt{M_x M_y E_x E_y}\cos\theta \qquad ...(3.52)$$

Equation (3.52) is quadratic in $z = \sqrt{E_y}$ so that we can write

$$az^2 + bz + c = 0 \qquad ...(3.53)$$

where $a = 1 + \frac{M_y}{M_Y}$, $\quad b = -(2/M_Y)\sqrt{M_x M_y E_x}\cos\theta$

and $\quad c = -E_x(1 - M_x/M_Y) - Q$

Eq. (3.53) has the solution

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad ...(3.54)$$

Written explicitly we then get

$$\sqrt{E_y} = \frac{1}{M_Y + M_y}\left\{(M_x M_y E_x)^{1/2}\cos\theta\right\} \pm \left[M_x M_y E_x \cos^2\theta + (M_Y + M_y)\left\{QM_Y + E_x(M_Y - M_y)\right\}\right]^{1/2} \qquad ...(3.55)$$

If we write $Q' = -Q$ then for endoergic reactions, $Q' > 0$ since $Q < 0$. In this case if $E_x = 0$, we have

$$b = 0 \text{ and } c = -Q = Q' > 0$$

The solution for z in this case becomes

$$z = \sqrt{E_y} = \pm\frac{\sqrt{-4ac}}{2a} = \pm\sqrt{-Q'/a}$$

Since both a and Q' are positive $z = \sqrt{E_y}$ is imaginary in this case. This means that the reaction is not possible with $E_x = 0$. A minimum energy $E_x = E_{\min}$ is needed to initiate endoergic reaction. In this case the term under the square root sign in Eq. (3.54) must be zero so that we get

$$b^2 - 4ac = 0$$

Substituting for a, b and c, we get

$$\frac{4}{M_Y^2}(M_x M_y E_{\min})\cos^2\theta = 4\left(1+\frac{M_y}{M_Y}\right)\left\{-Q - E_{\min}\left(1-\frac{M_x}{M_Y}\right)\right\}$$

which gives

$$E_{\min} = -\frac{(M_y + M_Y)Q}{M_y + M_Y - M_x - (M_x M_y / M_Y)\sin^2\theta} \quad ...(3.56)$$

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Since $Q < 0$, $E_{\min} > 0$, we get

$$E_{\min} = \frac{-(M_y + M_Y)Q}{M_y - Q - (M_x M_y / M_Y)\sin^2\theta} \quad ...(3.57)$$

$E_{\min}$ depends on the angle at which the particle y is emitted. When $\theta = 0$, *i.e.*, y is emitted in the forward direction, $E_{\min}$ has the lowest value and is known as the *threshold energy* for the endoergic reaction and is usually written as E_{th}. From Eq. (3.57) we get

$$E_{th} = \frac{(M_y + M_Y)Q}{M_x - Q} \quad ...(3.58)$$

Since $Q << M_x$, we can neglect it in the denominator of Eq. (3.58). Also we can replace $M_y + M_Y$ in the numerator by $M_x + M_X$. So we get finally

$$E_{th} \approx -Q.\frac{M_x + M_X}{M_X} = -Q\left(1 + \frac{M_x}{M_X}\right) \quad ...(3.59)$$

So by measuring the minimum energy E_{th} at which an exoergic reaction is initiated it is possible to determine the Q value of the reaction.

An inspection of Eq. (3.55) shows that under certain circumstances E_y will be a double-valued function of the projectile energy E_x *i.e.*, for a given E_x, there may be two values of E_y, the energy of the emitted particle. This happens only for endoergic reactions. The double valued nature of E_y is revealed in Fig. 3.13 for the $^3H(p, n)\,^3He$ endoergic reaction which has $Q = -0.7638$ MeV. Eq. (3.55) also shows that E_y is single valued if the following condition is satisfied:

$$QM_Y + E_x (M_Y - M_x) \geq 0$$

or,
$$E_x \geq \frac{-QM_Y}{M_Y - M_x}$$

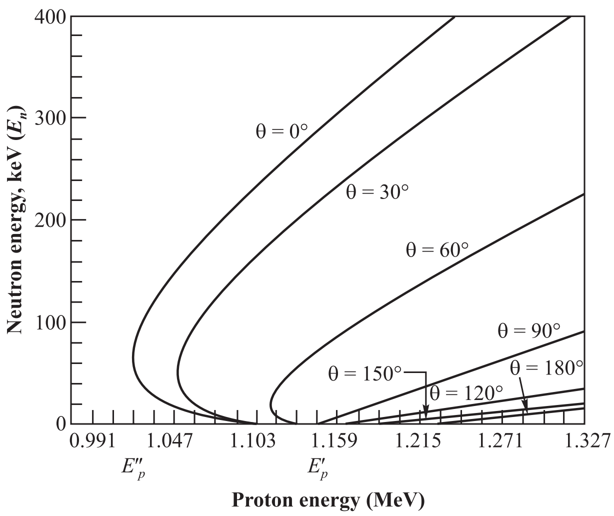

Fig. 3.13 *En versus Ep graph in 3H(p, n) 3He reaction. Double valued nature of neutron energy should be noted.*

Thus there is a limiting energy of the projectile above which the emitted particle energy will be single valued. This is given by

$$E'_x = -\frac{QM_Y}{M_Y - M_x} \qquad ...(3.60)$$

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For the case cited above $E'_x = 1.145$ MeV. For projectile energy greater than E'_x, the product particle y can be emitted at all angles between 0° and a maximum angle θ_{max}, which can be found with the help of Eq. (3.55).

3.10 NUCLEAR REACTION

From the beginning of civilization, people in different parts of the world had an intense desire to know whether baser metals like iron, copper etc., could be transformed into the noble metals like gold or silver. There were many who thought that such transformation was possible. In the middle ages, a pseudo-science known as *alchemy*, had flourished in Europe. Alchemists claimed that they could transform baser metals into noble metals, though there was little scientific basis for their claim. In fact many of them had to pay dearly for their fraudulent activities.

The discovery of radioactivity at the beginning of the present century led to the realisation that the radioactive elements spontaneously transformed into other elements. Following this discovery, the ancient dream of the alchemists was again revived in the minds of scientists regarding the possibility of transforming one element into another.

From our knowledge about the structure of the atomic nuclei it is clear that if we can change the number of protons or neutrons or both inside the nucleus, then it would be possible to bring about a transformation of the nucleus. If the proton number Z is changed, then it is possible to transform one element into another. On the other hand, if the neutron number N is changed, then one isotope of an element will be transformed into another isotope of the same element.

The main difficulty in producing the transformation of a nucleus artificially is the very tight binding of the nucleons inside the nucleus. To remove a nucleon from a nucleus, we must supply it a quantity of energy at least equal to the energy of its binding within the nucleus, which is usually of the order of a few MeV. This energy can be supplied by introducing a nuclear particle (*e.g.*, a proton, neutron, deuteron or an α-particle) into the nucleus from outside. Except neutrons, all the others are positively charged and hence are strongly repelled by the positive charge of the nucleus. So they must be highly energetic to be able to enter the nucleus to bring about a nuclear transformation.

Lord Rutherford was the first to produce artificial transformation (transmutation) of a nucleus in 1919, using the highly energetic α-particles from naturally radioactive substances like radium as projectiles.

Note: It may be mentioned that as early as 1916, the Indian physicist D.M. Bose working in the laboratory of Regener in Germany found in a cloud chamber photograph, the evidence for the emission of a charged particle from the end of an α–track with a range much longer than the range of the α–particle in the gas fillingthe chamber. Another shorter track which was much thicker also came out from the same point. It was clearly the case of an α–induced nuclear transformation which, however, could not be recognised as such by Bose.

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The apparatus used by Rutherford is shown in Fig. 3.14.

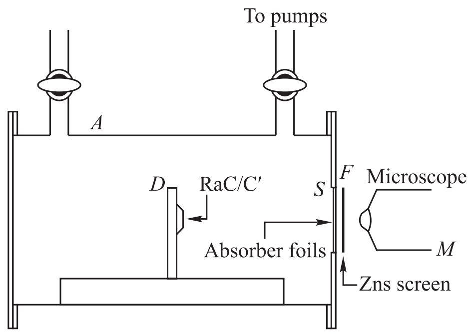

Fig. 3.14 *Rutherford's apparatus for producing artificial disintegration of nuclei.*

An air-tight glass chamber A which could be evacuated with the help of a vacuum pump and then filled with any desired gas, contained a small sample D of a naturally radioactive substance.

α-particles from the source D travelled through the gas in the chamber towards a thin window covering a port at the other end on the chamber wall. Outside the window, there was a fluorescent screen F on which scintillations were produced by the energetic charged particles falling on it. Thin metallic absorber foils S could be interposed between the window and F. The scintillations could be observed with the help of a microscope M.

The distance from D to the window was kept greater than the range of the α-particles from the source in the gas within the chamber. No scintillation could be observed when the chamber was filled with CO_2 or oxygen. However, when the chamber was filled with dry air or nitrogen, scintillations could be observed, even when the distance between the source and the screen F was 40 cm or more air-equivalent.

Rutherford identified the particles producing the scintillations as protons by deflecting them by a magnetic field. Their much longer range compared to that expected for the elastically scattered protons from hydrogen gas (28 cm) excluded the possibility of their origin from any hydrogen gas which might be mixed with nitrogen as impurity.

Rutherford explained his observations in the following way. When the very high velocity α-particles made head-on collisions with the nitrogen nuclei ^{14}N some of them were captured by the latter. The composite system, which was formed as a result of such capture, almost immediately (within ~ 10^{-15} s) disintegrated by the emission of a proton of very high velocity. This was the process of *nuclear transmutation* brought about artificially with the help of α-particles from a radioactive substance, leaving a residual nucleus of the isotope ^{17}O of oxygen. The process can be represented by means of an equation analogous to the equation for a chemical reaction as follows:

$$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{18}_{9}F^{*} \rightarrow ^{17}_{8}O + ^{1}_{1}H \qquad ...(3.61)$$

The intermediate step ^{18}F is known as a compound nucleus. It breaks up almost immediately after its formation. In writing such a nuclear reaction equation, we often omit this intermediate step and write only the initial and final steps in the process.

A nuclear reaction refers to a process which occurs when a nuclear particle (*e.g.*, a nucleon, a nucleus or an elementary particle) comes into close contact with another during which energy and momentum exchanges take place. The final products of the reaction are again some nuclear particle or particles which leave the point of contact (reaction site) in different directions. The changes produced in a nuclear reaction usually involve strong nuclear force. Purely electromagnetic effects (*e.g.*, Coulomb scattering) or proceses involving weak interactions (*e.g.*, β-decay) are usually excluded from the category of nuclear reaction. However, changes of nuclear states under the influence of electromagnetic interactions are included.

In general, a nuclear reaction can be represented by an equation in the following form:

$$^{A}_{Z}X + x \rightarrow {}^{A'}_{Z'}Y + y \qquad ...(3.62)$$

or simply as $^{A}X\ (x, y)\ ^{A'}Y$.

Here X is the target nucleus which is bombarded by the projectile *x*. The resulting compound nucleus breaks up almost immediately by ejecting a particle *y*, leaving a residual nucleus Y. Since the chemical symbol of the atoms indicates their atomic numbers (Z), these are often omitted in writing the nuclear reaction equation. The projectile *x* and the emitted particle *y* are in many cases light nuclei such as protons (*p*), neutrons (*n*), deuterons (*d*), α-particles (α), γ-rays (γ) etc. and in the nuclear reaction equations, these symbols are generally used.

Types of Nuclear Reactions

The artificial transmutation of a nucleus produced in the pioneering experiment of Rutherford is a type of nuclear reaction. Various types of nuclear reactions have since been produced. These can be conveniently classified as below:

(*a*) *Elastic scattering*: In this case the ejected particle *y* is the same as the projectile *x*. It comes out with the same energy and angular momentum as *x*, so that the residual nucleus Y is the same as the target X and is left in the same state (ground state) as the latter. We can represent the process as X (*x*, *x*) X.

(*b*) *Inelastic scattering*: In this case *y* is the same as *x*. But it has different energy and angular momentum, so that the residual nucleus *y* (= X) is left in an excited state. The process can be written as X (*x*, *y*) X*, where the asterisk on X indicates an excited state of X.

(*c*) *Radiative capture*: In this case the projectile *x* is absorbed by the target nucleus X to form the excited compound nucleus (C*) which subsequently goes down to the ground state by the emission of one or more γ-ray quanta. We can write the process as X (*x*, *y*) Y*, (Y = C).

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(*d*) *Disintegration process*: We can represent the process as X (x, y) Y where X, x, Y and y are all different either in Z or in A or in both. The first nuclear transmutation observed by Rutherford is an example of this process: ^{14}N (α, p) ^{17}O.

(*e*) *Many body reaction*: When the kinetic energy of the incident particle is high, two or more particles can come out of the compound nucleus. If y_1, y_2, y_3, etc. represent these different particles, we can write the reaction equation as X (x, y_1, y_2, y_3 ...) Y. Examples are ^{16}O (p, 2p) ^{15}N; ^{16}O (p, pn) ^{15}O, ^{16}O (p, 3p) ^{14}C etc. When the energy of x is very high, a very large number of reaction products usually result (3 to 20 for example). Such reactions are known as spallation reactions.

(*f*) *Photo-disintegration*: In this case the target nucleus is bombarded with very high energy γ-rays, so that it is raised to an excited state by the absorption of the latter. The compound nucleus C* = X*. The reaction can be written as X (γ, y) Y.

(*g*) *Nuclear fission*: When X is a heavy nucleus and y, Y have comparable masses, the reaction is known as nuclear fission. An example is ^{238}U (n, f).

(*h*) *Elementary particle reactions*: These involve either the production of elementary particles other than nucleons or nuclei as a result of the reaction or their use as projectiles or both of these. Examples are:

$$p + p \rightarrow p + n + \pi^+;$$
$$\pi^- + p \rightarrow \pi^0 + n;$$
$$p + \pi^0 \rightarrow K^0 + \wedge^0 \text{ etc.}$$

These reactions are usually produced at extremely high energies which may be several hundred MeV or more.

(*i*) *Heavy ion reactions*: In these reactions the target nucleus is bombarded by projectiles heavier than α-particles. Various types of products may be produced. The reactions usually take place at fairly high energies (several hundred MeV) of the projectile. Examples are:

^{10}B (^{16}O, ^{4}He) ^{22}Na,

^{14}N (^{14}N, ^{15}N) ^{13}N etc.

3.10.1 Cross Section and Level Width

The probability of the occurance of a nuclear reaction is measured by the *reaction cross section*. It is usually designated by the symbol σ. The cross section of a nuclear reaction X(x, y)Y can be written as $\sigma(x, y)$. If a parallel beam of N projectiles is incident in a given interval of time upon a target foil T of thickness Δx and surface area S normally, then the number of nuclei in T undergoing transformation due to the reaction of the type under consideration, is proportional to the intensity of the incident beam of projectiles and to the total number of target nuclei present in the foil [Refer Figure 3.15a). The incident particle intensity is (N/S) and the number of nuclei present in the foil is ($n\,S\,\Delta\,x$). So the number of nuclei transformed is

$$\Delta\,N \propto (N/S)\,(nS\,\Delta\,x)$$

or,
$$\Delta\,N = \sigma N n\,\Delta x = \sigma N n_1 \qquad ...(3.63)$$

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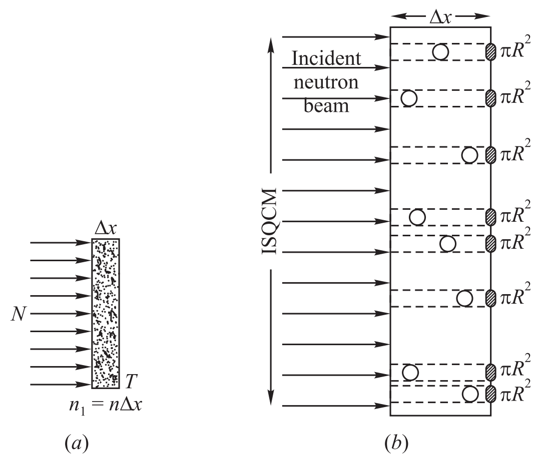

Fig. 3.15 *(a) Bombardment of target foil (T) by a beam of particles. (b) Geometrical significance of reaction cross section.*

Here $n_1 = n\,\Delta x$ is the number of target nuclei per unit area of the foil, n being the number of nuclei per unit volume. Eq. (3.63) shows that since both ΔN and N are pure numbers and $n_1 = n\,\Delta x$ has the dimension of the reciprocal of an area, σ has the dimension of an area. Hence it is called the cross section and measures the probability of the occurrence of the reaction when a single particle ($N = 1$) falls on a single target nucleus present per unit area ($n_1 = 1$). Since the nuclear radii are of the order of 10^{-14} to 10^{-15} m, the cross section of the nuclear reaction is of the order of 10^{-28} m². The commonly used unit of the nuclear reaction cross section is a *barn*:

$$1 \text{ barn} = 10^{-28} \text{ m}^2$$

Though the cross sections for most nuclear reactions are of the order of a few barns or even less, they may be very high (several thousand barns) for some special types of reactions, such as the (n, γ) reaction induced by thermal neutrons or the neutron-induced reasonance reactions.

The geometrical significance of the reaction cross section can be understood in the following manner. Referring to Fig. 3.15(b) we see that if R is the effective radius of the target nucleus for a given reaction, then the projection of its surface area on a plane perpendicular to the direction of motion of the projectile, shown shaded in the figure is πR^2. So the number of projectiles encountering each target nucleus is $\pi R^2 N_s$ where $N_s = N/S$ is the number of projectiles incident per unit area of the target. The projectiles are assumed to be mass–points. Since there are n_1 nuclei per unit area of the target, the number of projectiles intercepted by the target nuclei in the foil is

$$n_1 S \times \pi R^2 N_s = \pi R^2 N n_1 \qquad \text{...(3.64)}$$

where $N = N_s \times S$ is the total number of projectiles incident on the target. Hence the probability of encounter between a single projectile ($N = 1$) with one nucleus per unit area ($n_1 = 1$) in the target foil is

$$\frac{\pi R^2 N n_1}{n_1} = \pi R^2 N = \pi R^2 \qquad \text{...(3.65)}$$

Actually the probability σ of encounter between a single projectile and a single target nucleus per unit area is not determined by πR^2 alone. This

NOTES

probability depends on the nature of the interaction between the projectile and the target nucleus, the energy of the projectile and other factors. Besides, the incident particle is not a mass–point as assumed above. So the reaction cross section depends on its size also. For very low energy projectiles, the de Broglie wavelength $\lambda = h/p$ is much longer than their geometrical extension, so that the region over which they interact is much larger than their geometrical cross section. This is the reason for the cross section of the (n, γ) reaction with thermal neutrons to be usually very large as stated aboye.

In the case of charged particles, the cross section is considerably reduced because of the strong electrostatic repulsion of the target nucleus.

In the above discussions, it has been assumed that the total projected area of all the nuclei in the foil which is $(\pi R^2 n_1 S)$ is small compared to the area S of the foil. This is true only if the foil thickness is small.

3.11 BOHR COMPOUND NUCLEUS THEORY OF NUCLEAR REACTION

We have talked about the formation of a compound nucleus as an intermediate step when a nuclear reaction takes place. The primary evidences on which this compound nucleus idea was developed came after the discovery of the neutron and its use as a projectile in producing nuclear reactions, from 1935 onwards. It was observed that for high energy neutrons, the total cross section for neutron absorption and scattering was of the order of πR^2 where R is the nuclear radius. For very low energies however, the cross section is higher and approaches the limiting value of $\pi ƛ^2$, $ƛ$ being the reduced de Broglie wavelength of the neutrons.

These results were sought to be explained by assuming that the incident neutron moved in an average potential well due to all the nucleons in the nucleus (Refer Fig. 3.16) for an interval of time which is of the order of

$$t \sim \frac{\text{Nuclear diameter}}{\text{Neutron velocity}} \sim \frac{10^{-14}}{10^{7}} = 10^{-21} \text{ s}$$

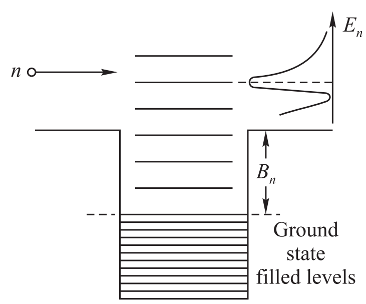

Fig. 3.16 *Motion of an incident neutron in an average single particle potential well. Bn is the neutron binding energy.*

In this case the incident neutron would have a large probability of escaping from the nucleus without absorption. So the elastic scattering cross section σ_{sc} should be large while capture or reaction cross section σ_{re} would be quite low in this picture. At very low energies, the capture cross section should however be relatively large, since the neutron would spend a longer time near the target nucleus. σ_{re} should depend on $1/v$ in this case. So at thermal energies σ_{sc} and σ_{re} should be comparable since both of these approach the limiting value of $\pi\bar{\lambda}^2$.

The energy levels in this single particle potential well should be well separated from one another, the separation being of the order of 5 to 10 MeV. There would also be some levels above the neutron separation energy S_n (virtual levels). The width of the level which measures the probability of its decay would be

$$\Gamma = \hbar/\tau = \hbar/10^{-21} \approx 1 \text{ MeV}$$

showing the levels to be quite broad. When the energy of the incident neutron corresponds to the excitation energy of one of the nuclear levels, resonances would be expected to be observed in the cross section *vs.* energy graph (see Fig. 3.16). These resonances should be widely spaced (several MeV) having large widths (~ MeV). Obviously at very low energies (near thermal energies) no such resonances would be expected, since neutrons of a few electron volts energy cannot be expected to produce resonances corresponding to levels with gaps of several million electron volts.

However such a picture does not agree at all with the observed results on neutron induced reactions. In many cases, the neutron absorption cross sections are found to be very large at thermal neutron energies while the elastic scattering cross sections are much lower. It may be noted that at these very low energies, the absorption cross section is due to the radiative capture, *i.e.*, (n, γ) type of reaction. Resonances are observed in most nuclei for both elastic scattering cross section (σ_{sc}) and radiative capture cross section $\sigma(n, \gamma)$. These resonances mostly appear at neutron energies between 0.1 to 10 eV, *i.e.*, they are very closely spaced. They are also found to be very sharp, having widths of the order of 0.1 eV or lower. These observations led Niels Bohr to propose the following mechanism for nuclear reactions (1936) which is known as the *compound nucleus hypothesis*.

When a nuclear projectile x enters into a target nucleus X to produce a nuclear reaction, an intermediate stage is formed before the production of the final product nuclei Y and y (Refer Equation 3.62):

$$\text{X} + x \rightarrow \text{C}^* \rightarrow \text{Y} + y$$

The incoming projectile x quickly dissipates its energy as it enters into the nucleus X and merges with the closely packed nucleons in it. As a result, the general random motion of all the nucleons in the nucleus is disturbed, each nucleon gaining some additional energy. But no single one of them will generally gain enough energy to enable it to come out of the nucleus which is of the order of a few million electron volts. However, after a relatively long time when a very large number of collisions among the nucleons have taken place (which may be of the order of 10 million), enough energy may be concentrated on one of the nucleons enabling it to escape from the nucleus which then deexcites (cools off) to the ground state. It may also be deexcited by the alternative process of emission of γ-rays. The whole process is similar

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to the heating of a drop of liquid containing large number of molecules. When enough energy is concentrated on some of the molecules, evaporation takes place to cool off the drop. The process of emission of a nucleon (or a group of nucleons) from the excited nucleus as mentioned above is thus similar to the phenomenon of evaporation. The analogy between the two systems was one of the points which led to the formulation of the liquid drop model of the nucleus.

The composite system that is formed as a result of the absorption of the incident particle *x* by the nucleus X is known as the *compound nucleus*. Though it ultimately breaks up by the emission of a particle *y* or of a γ-ray, it lives long enough compared to time taken by a nucleon of a few MeV energy to travel through the mean free path of collision between the nucleons in the nucleus (which is somewhat less than the nuclear radius, but is of the same order of magnitude). The mean time between collisions is about $(2 \times 10^{-15}/5 \times 10^{7})$ or $\sim 10^{-22}$ s. So the life of the compound nucleus is of the order of

$$t \sim 10^{7} \times 10^{-22} = 10^{-15} \text{ s}$$

It may be noted that the mean time for radiative transitions within the nuclei ($\sim 10^{-13}$ s) is much longer than the time for decay of the compound nucleus by particle emission.

The particle *y* emitted in the decay of the compound nucleus is generally different from the particle *x* which enters X to produce the compound nucleus. In the event of *y* being identical with *x*, we have inelastic scattering. The residual nucleus Y = X* in this case is the same as the target nucleus produced in a different energy state. In the rare case in which the residual nucleus is identical with the target nucleus and is produced exactly in the same state as the latter, we get what is known as the *compound elastic* or *resonance scattering*. As stated before, elastic scattering may alternatively take place by the action of the nuclear potential on the incident particle without the entry of *x* into X to form the compound nucleus. This is known as the *potential scattering* and has a much greater probability than the other.

Since the compound nucleus is a relatively longer lived entity, the nuclear reaction actually proceeds in two steps: (*i*) the formation of the compound nucleus by the absorption of the incident particle by the target nucleus. (*ii*) the disintegration of the compound nucleus in a manner which is independent of the method of its formation into the reaction products *y* and Y in definite quantum states. Sometime, the residual nucleus Y may be left in a highly excited state which may then "boil off", another particle *y*′ to leave the nucleus Y′ leading to a two particle emission process: X (*x*, *yy*′) Y′. The process may continue and another particle *y*″ may be emitted from the excited Y′ leaving the residual nucleus Y″ in the third stage. Thus a series of particles (usually neutrons) may be boiled off successively from a highly excited nucleus.

The two stages by which a nuclear reaction proceeds may be written symbolically as

NOTES

$$X + x \rightarrow C^* \rightarrow Y + y$$
$$\rightarrow C + y$$

According to Bohr, the two stages, *viz.*, the formation of the compound nucleus and its break up are independent. This is known as the *independence hypothesis*. The decay of C* depends only on the properties of C* and not upon how it was formed. In other words since C* is a relatively long lived entity, by the time it is ready to break up, *it forgets as to how it had been formed*.

The probability of decay is equal to the reciprocal of the mean-life τ of the compound nucleus. If Γ is the width of the level, we can use the uncertainty relation to write

$$\Gamma \times \tau \sim \hbar$$

which gives $$\Gamma = \hbar/\tau$$

Thus the width of the level is a measure of the probability of its decay. Actually the compound nucleus may decay by the emission of different types of particles of y, y', y'' etc., leaving a different residual nucleus in each case. Each of these has a different probability of occurrence.

If Γ_y is the *partial width* of the level for decay by the emission of y, then considering the various possible types of decay, we get the total width of the level as

$$\Gamma = \sum_y \Gamma_y + \Gamma_\gamma = (\Gamma_y + \Gamma_{y'} + \Gamma_{y''} + ...) + \Gamma_\gamma \quad ...(3.66)$$

The relative probabilities of the different types of decay are then

$$\eta_y = \Gamma_y/\Gamma, \ \eta_{y'} = \Gamma_{y'}/\Gamma, \ ... \ \eta_\gamma = \Gamma_\gamma/\Gamma \quad ...(3.67)$$

Because of the independence hypothesis, we can write the cross section for the process X(x, y)Y as the product of the cross section σ_x for the formation of the compound nucleus and the probability of its decay:

$$\sigma(x, y) = \sigma_x \eta_y = \sigma_x \Gamma_y/\Gamma \quad ...(3.68)$$

The above way of writing the cross section implies that only one particular energy state of C* is being considered (*i.e.*, only one resonance). This is possible if the levels are well separated and are so sharp that they do not interfere with one another. In other words, the mean level spacing $D >> \Gamma$. We shall first consider such isolated levels. The case of overlapping levels will be considered later.

3.11.1 Deuteron Stripping Reaction

In nuclear physics, a stripping reaction is a nuclear reaction in which part of the incident nucleus combines with the target nucleus, and the remainder proceeds with most of its original momentum in almost its original direction. This reaction was first described by Stuart Thomas Butler in 1950. Deuteron stripping reactions have been extensively used to study nuclear reactions and

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structure, this occurs where the incident nucleus is a deuteron and only a proton emerges from the target nucleus. A simple one-step stripping reaction can be represented as

$$A + a \rightarrow B + b$$

$$A + (b+x)_a \rightarrow (A+x)_b + b$$

where A represents the target core, b represents the projectile core, and x is the transferred mass which may represent any number of particles

3.12 BREIT-WIGNER SINGLE LEVEL FORMULA

To investigate nuclear reactions, a quantitative assessment of the likelihood of a given nuclear reaction is required. This number must be experimentally quantifiable and calculated in such a way that theoretical and experimental values can be easily compared. The quantity most frequently used for this purpose is the nucleus cross section for a given reaction, which is commonly represented by σ with the appropriate subscript. Nuclear cross section is easily viewed as the cross-sectional area or target area that the nucleus presents to an incident particle.

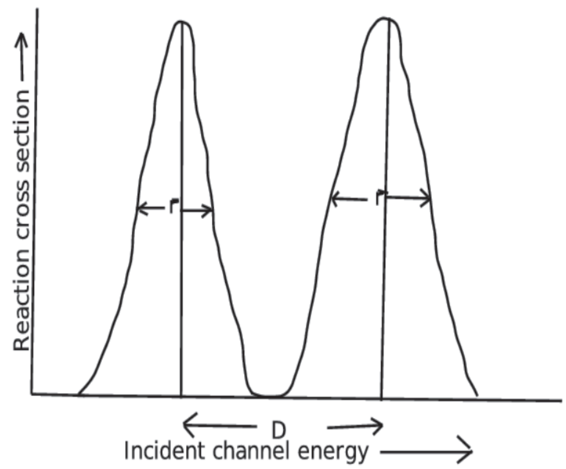

Fig. 3.17 *Reaction cross section as a function of incident channel energy.*

A nuclear reaction caused by the absorption of a projectile x by a target nucleus X (both in their ground state) that results in the formation of a compound nucleus C* in an excited state close to one of the latter's isolated levels that is far removed from any of its other levels. The presence of such an isolated level means that the separation between them is D >> Γ; where Γ is the level's width. The reaction is begun by a distinct entrance channel ($X + x$) defined by a distinct kinetic energy of relative motion Ex between X and x and a distinct relative angular momentum.
The energy required to excite the compound nucleus in this state is given by

$$E_c = E_x + S_x = Er \quad \text{…..(3.69)}$$

Where Er denotes the energy of the isolated level during the formation

NOTES

of the compound nucleus. *Sx* is the separation energy of *x* from the ground state of the compound nucleus, denoted by

$$S_x = B_c - B_X - B_x \qquad \text{.....(3.70)}$$

The binding energies of the corresponding nuclei are denoted by the *B*. *Bx = 0* when x is a nucleon. With the relative kinetic energy *Ey*, the complex nucleus is broken up into *Y+y*. Obviously, *Ec = Ey + Sy* can also be written, where *Sy* is the separation energy of *y* from the compound nucleus in the ground state, which is given by.

$$S_y = B_c - B_Y - B_y$$

By = 0; if *y* is a nucleon. Both *Y* and *y* are assumed to be produced in their ground states. However, this is not always the case, as *Y* may be left in various excited states, resulting in a variety of exit channels.

A damped harmonic wave can be used to indicate the condition of the compound nucleus formed as described above;

$$\Psi(t) = \psi_o \exp(-iE_r\, t/\hbar) \exp(-\Gamma t/2\hbar)$$

$$= \Psi_o \exp\left\{-i\left(E_r - \frac{i\Gamma}{2}\right)t/\hbar\right\} \qquad \text{......(3.71)}$$

Here Γ/2 denotes the half width of the level, which is essentially a decaying condition with a life – time of $\eta = \hbar / \Gamma$.

The above wave function does not represent a stationary state, but may be constructed using the Fourier integral approach from the superposition of stationary states of various energies.

$$\psi(t) = \int_{-\infty}^{+\infty} A_E \exp(-iEt/\hbar) dE \qquad \text{......(3.72)}$$

By doing the Fourier transform of equation (4), we can determine the amplitude *AE* of the state at energy *E*.

$$A_E = \frac{1}{2\pi}\int_0^{\infty} \psi(t^1) \exp(iEt'/\hbar) dt'$$

$$= \frac{1}{2\pi}\int_0^{\infty} \Psi_o \exp\{i(E - E_r + i\Gamma/2)t'/\hbar\} dt'$$

$$= \frac{\psi_o}{2\pi}\left[\frac{\exp\{i(E - E_r + i\Gamma/2)t'/\hbar\}}{i(E - E_r + i\Gamma/2)/\hbar}\right]_0^{\infty}$$

Only positive values of time are used here, as the composite nucleus can decay only after it is formed.

$$\therefore A_E = \frac{\psi_o}{2\pi}\frac{i\hbar}{(E - Er + i\Gamma/2)}$$

NOTES

Due to the damping term exp($-\Gamma t'/2\hbar$), the upper limit of the preceding integral vanishes. Thus, we obtain

$$|A_E|^2 = \frac{|\psi_o|^2}{4\pi^2}\frac{\hbar^2}{(E-E_r)^2+\Gamma^2/4}$$

The cross section for forming the state E_c is proportional to the amplitude squared by the process $X + x$. As a result, we can write.

$$\sigma_x = \frac{C}{(E-E_r)^2+\Gamma^2/4}$$

In this case, C is a constant value. To determine C, we note that the incident channel's total number of possible states is,

$$dn = \frac{4\pi p_x^2 \Omega dp_x}{(2\pi\hbar)^3}$$

where Ω is the volume of the enclosure within which the reaction take place. If σ_x is the cross section for the absorption of x by X, then the volume swept out by the effective collision area in one second is $\sigma_x v_x$ where v_x is the relative velocity of the incident particle. So, the probability of finding the nucleus X in this volume is $\sigma_x v_x/\Omega$ and the probability of formation of the compound nucleus in the given entrance channel per second is

$$\frac{\sigma_x v_x}{\Omega}\cdot\frac{4\pi p_x^2 \Omega dp_x}{(2\pi\hbar)^3} = \frac{\sigma_x p_x^2 v_x dp_x}{2\pi^2\hbar^3} = \frac{\sigma_x p_x^2 v_x dE_x}{2\pi^2\hbar^3}$$

We obtain the entire probability by integrating across all potential energies;

$$P = \frac{1}{2\pi^2\hbar}\int_{-\infty}^{+\infty}\frac{\sigma_x}{\lambda\!\!\!^{-2}}dE_x$$

We may ignore the variation ƛ and write ($dE = dE_x$) since the integrand has finite values only for the energies within the width Γ of the level, which are extremely narrow.

$$P = \frac{1}{2\pi^2\hbar\lambda\!\!\!^{-2}}\int_{-\infty}^{+\infty}\sigma_x dE$$

$$= \frac{C}{2\pi^2\hbar\lambda\!\!\!^{-2}}\int_{-\infty}^{+\infty}\frac{dE}{(E-E_t)^2+\Gamma^2/4}$$

$$= \frac{C}{2\pi^2\hbar\lambda\!\!\!^{-2}}\cdot\frac{2\pi}{\Gamma} = \frac{C}{\pi\hbar\Gamma\lambda\!\!\!^{-2}}$$

The chance of the compound nucleus forming above must be the same as the likelihood of C* decaying along the same channel. The reciprocity theorem leads to this conclusion. We get if we represent this decay probability through the entrance channel as $\Gamma_x/\hbar$.

$$\frac{C}{\pi\hbar\Gamma\lambda\!\!\!^{-2}}\cdot\frac{\Gamma_x}{\hbar} \qquad \text{Or} \qquad C = \pi\lambda\!\!\!^{-2}\Gamma_x\Gamma$$

As a result, the cross section for the production of the compound nucleus is as follows:

$$\sigma_x = \frac{\pi\lambda\!\!\!^{-2}\,\Gamma_x \Gamma}{(E - E_t)^2 + \Gamma^2/4}$$

Γ_y/Γ represents the relative likelihood of C* decaying through the exit channel $Y + y$. The cross section for the reaction $X(x,y)Y$ is then calculated as

$$\sigma(x, y) = \sigma_x \frac{\Gamma_y}{\Gamma} = \pi\lambda\!\!\!^{-2} \frac{\Gamma_x \Gamma_y}{(E - E_r)^2 + \Gamma^2/4}$$

For spinless nuclei at very low energies, this is the Breit Wigner one-level formula, where the relative angular momentum of the particles in the entrance channel is $l = 0$. If l is not zero, as it is when the energy is larger, we must account for the statistical factor of the compound state created, which is given by $g = 2\,l + 1$ for spinless nuclei x and X.

Each of the (2 l +1) sub states have an equal chance of decaying. As a result, Γ_x must be multiplied by this factor, which yields.

$$\sigma_x^l = \pi\lambda\!\!\!^{-2}\,(2l + 1)\frac{\Gamma_x \Gamma}{(E - E_r)^2 + \Gamma^2/4}$$

$$\sigma^{(l)}(x, y) = \pi\lambda\!\!\!^{-2}\,(2l + 1)\frac{\Gamma_x \Gamma_y}{(E - E_r)^2 + \Gamma^{2.}/4}$$

Check Your Progress

6. Name the conservation laws of nuclear reactions.
7. To what is the discovery of radioactivity at the beginning of the present century lead?
8. State the Bohr compound nucleus theory of nuclear reaction.
9. What is stripping reaction?

3.13 ANSWERS TO 'CHECK YOUR PROGRESS'

1. The liquid drop model was first proposed by N. Bohr and F. Kalckar in 1937 and was later applied by C.F. von Weizsäcker and H.A. Bethe to develop a semi-empirical formula for the binding energy of the nucleus.
2. Each individual molecule within a liquid drop exerts an attractive force upon a group of molecules in its immediate neighbourhood. The force of interaction does not extend to all the molecules within the drop. This is known as the *saturation* of the force.

NOTES

3. The liquid drop model is not very successful in describing the low lying excited states of the nucleus. Because of the collective motions of the large number of nucleons involved, the model gives rise to closely spaced energy levels. Actually however, these are found to be quite widely spaced at low excitation energies.
4. The drop's potential energy at each stage can be estimated as a function of its degree of deformation. The potential energy is displayed versus r, the distance between two fission fragments' centers.
5. In order to explain the disagreement at the higher magic numbers, Mayer and independently Haxel, Jensen and Suess suggested that a *spin-orbit interaction* term should be added to the central potential $V(r)$

$$Q_{sp} = -\frac{2j-1}{2j+2}\langle r^2 \rangle$$

   where $\langle r^2 \rangle$ is the mean square radius of the charge distribution which in the present case is equal to the mean square distance of the proton from the nuclear centre.
6. The conservation laws of nuclear reactions are:
   - Conservation of mass number
   - Conservation of atomic number
   - Conservation of energy
   - Conservation of linear momentum
   - Conservation of angular momentum
   - Conservation of parity
   - Conservation of isotopic spin
7. The discovery of radioactivity at the beginning of the present century led to the realisation that the radioactive elements spontaneously transformed into other elements. Following this discovery, the ancient dream of the alchemists was again revived in the minds of scientists regarding the possibility of transforming one element into another.
8. The primary evidences on which this compound nucleus idea was developed came after the discovery of the neutron and its use as a projectile in producing nuclear reactions, from 1935 onwards. It was observed that for high energy neutrons, the total cross section for neutron absorption and scattering was of the order of pR^2 where R is the nuclear radius. For very low energies however, the cross section is higher and approaches the limiting value of pD^2, ƛ being the reduced de Broglie wavelength of the neutrons.
9. In nuclear physics, a stripping reaction is a nuclear reaction in which part of the incident nucleus combines with the target nucleus, and the remainder proceeds with most of its original momentum in almost its original direction. This reaction was first described by Stuart Thomas Butler in 1950.

3.14 SUMMARY

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- The macroscopic properties of the nucleus e.g., the constant density of the nuclear matter and the constant binding energy per nucleon are very similar to those found in a liquid drop.

  The very strong short range interaction between the nucleons permits us to consider their collective behaviour in determining the properties of the nucleus.
- The liquid drop model was first proposed by N. Bohr and F. Kalckar in 1937 and was later applied by C.F. von Weizsäcker and H.A. Bethe to develop a semi-empirical formula for the binding energy of the nucleus.
- The attractive force near the nuclear surface is similar to the force of surface tension on the surface of the liquid drop.
- Different types of particles, *e.g.*, neutrons, protons, deuterons, a-particles etc. are emitted during nuclear reactions. These processes are analogous to the emission of the molecules from the liquid drop during evaporation.
- The internal energy of the nucleus is analogous to the heat energy within the liquid drop.
- The formation of a short lived compound nucleus by the absorption of a nuclear particle in a nucleus during a nuclear reaction is analogous to the process of condensation from the vapour to the liquid phase in the case of the liquid drop.
- The drop's potential energy at each stage can be estimated as a function of its degree of deformation. The potential energy is displayed versus r, the distance between two fission fragments' centers.
- Nuclear shell model is thought that protons and neutrons in a nucleus are constantly colliding with each other. With such a strong force acting between them and so many nucleons to collide with, nucleons cannot conceivably complete entire orbits without interacting.
- In order to explain the disagreement at the higher magic numbers, Mayer and independently Haxel, Jensen and Suess suggested that a *spin-orbit interaction* term should be added to the central potential $V(r)$
- The nucleus would require us to consider the motion of the individual nucleons in a potential well which would give rise to the existence of a nuclear shell structure, similar to the electronic shells in the atoms.
- In conservation of mass number the total number of neutrons and protons in the nuclei taking part in a nuclear reaction remains unchanged after the reaction.
- In conservation of atomic number the total number of protons of the nuclei taking part in a nuclear reaction remains unchanged after the reaction.

- The discovery of radioactivity at the beginning of the present century led to the realisation that the radioactive elements spontaneously transformed into other elements. Following this discovery, the ancient dream of the alchemists was again revived in the minds of scientists regarding the possibility of transforming one element into another.
- The primary evidences on which this compound nucleus idea was developed came after the discovery of the neutron and its use as a projectile in producing nuclear reactions, from 1935 onwards. It was observed that for high energy neutrons, the total cross section for neutron absorption and scattering was of the order of pR^2 where R is the nuclear radius. For very low energies however, the cross section is higher and approaches the limiting value of pD^2, ƛ being the reduced de Broglie wavelength of the neutrons.
- In nuclear physics, a stripping reaction is a nuclear reaction in which part of the incident nucleus combines with the target nucleus, and the remainder proceeds with most of its original momentum in almost its original direction. This reaction was first described by Stuart Thomas Butler in 1950.
- A quantitative assessment of the likelihood of a given nuclear reaction is required. This number must be experimentally quantifiable and calculated in such a way that theoretical and experimental values can be easily compared. The quantity most frequently used for this purpose is the nucleus cross section for a given reaction, which is commonly represented by σ with the appropriate subscript. Nuclear cross section is easily viewed as the cross-sectional area or target area that the nucleus presents to an incident particle.

3.15 KEY TERMS

- **Fission:** When a neutron collides with a larger atom, it causes it to excite and split into two smaller atoms, which are known as fission products. There are also more neutrons released, which can start a chain reaction. A great quantity of energy is produced when each atom divides.
- **Nuclear shell model:** Accordint to Nuclear shell model protons and neutrons in a nucleus are constantly colliding with each other. With such a strong force acting between them and so many nucleons to collide with, nucleons cannot conceivably complete entire orbits without interacting.
- **Conservation of mass number:** It means that the total number of neutrons and protons in the nuclei taking part in a nuclear reaction remains unchanged after the reaction.
- **Conservation of atomic number:** It states that the total number of protons of the nuclei taking part in a nuclear reaction remains unchanged after the reaction.

- **Stripping reaction:** A stripping reaction is a nuclear reaction in which part of the incident nucleus combines with the target nucleus, and the remainder proceeds with most of its original momentum in almost its original direction. This reaction was first described by Stuart Thomas Butler in 1950.

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3.16 SELF ASSESSMENT QUESTIONS AND EXERCISES

Short-Answer Questions

1. What is liquid drop model and fission?
2. What do you understand by the spin orbit coupling?
3. What is quadrupole moments?
4. Define nuclear shell structure.
5. What do you mean by the conservation of mass number and atomic number?
6. How will you define the threshold energy of a nuclear reaction?
7. Define nuclear reaction, cross section and level width.
8. State the Bohr compound nucleus theory of nuclear reaction.
9. What is deuteron stripping reaction?
10. Write the Breit-Wigner single level formula.

Long-Answer Questions

1. Discuss about the liquid drop model and fission with the help of examples.
2. Explain the Bohr and wheeler's theory.
3. Describe the spin orbit coupling.
4. Explain the magnetic and quadrupole moments with the help of relevant examples.
5. Explain the nuclear shell structure and elementary idea of collective model of the nucleus.
6. Discuss about the conservation laws of nuclear reactions and Q value.
7. What do you understand by the threshold energy of a nuclear reaction? Explain.
8. Explain the nuclear reactions, cross section and level width with the help of giving examples.
9. Describe the deuteron stripping reactions.
10. Illustrate the Breit-Wigner single level formula with the help of giving examples.

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3.17 FURTHER READING

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UNIT 4 NUCLEAR DECAY

NOTES

Structure

4.0 Introduction
4.1 Objectives
4.2 Beta Ray Spectrum: Decay and Nature
4.3 Neutrino Hypothesis
4.4 Fermi Theory of Beta Decay: Allowed and Forbidden Transitions Parity Violations in Beta Decay
4.5 Concept of Helicity
4.6 Multipole Transition and Selection Rules for the Decay of the Nuclei Selection Rules
4.7 Internal Conversion, Conversion Coefficients of Isomeric Nuclei
4.8 Angular Correlation of Successive Decay
4.9 Answers to 'Check Your Progress'
4.10 Summary
4.11 Key Terms
4.12 Self Assessment Questions and Exercises
4.13 Further Reading

4.0 INTRODUCTION

Nuclear decay or radioactive decay is the emission of energy in the form of ionizing radiation. The ionizing radiation that is emitted can include alpha particles, beta particles and/or gamma rays. Radioactive decay occurs in unbalanced atoms called radionuclides. **Nuclear decay** occurs when the nucleus of an atom is unstable and spontaneously emits energy in the form of radiation. The result is that the nucleus changes into the nucleus of one or more other elements. These *daughter nuclei* have a lower mass and are more stable (lower in energy) than the parent nucleus. Elements in the periodic table can take on several forms. Some of these forms are stable; other forms are unstable. Typically, the most stable form of an element is the most common in nature. However, all elements have an unstable form. Unstable forms emit ionizing radiation and are radioactive. There are some elements with no stable form that are always radioactive, such as uranium. Elements that emit ionizing radiation are called radionuclides. In this unit, we will study in detail about beta ray spectrum along with its decay and nature, hypothesis of neutrino creation, meaning of helicity, multipole transition and selection rules for the decay internal conversion and conversion coefficients of isomeric nuclei and the angular correlation of successive decay.

4.1 OBJECTIVES

After going through this unit, you will be able to:

- Explain beta ray spectrum along with its decay and nature
- Describe the hypothesis of neutrino creation
- State the meaning of helicity

- Discuss the multipole transition and selection rules for the decay
- Explain the internal conversion and conversion coefficients of isomeric nuclei
- Describe the angular correlation of successive decay

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4.2 BETA RAY SPECTRUM: DECAY AND NATURE

A radioactive nucleus's atomic number changes by one when it undergoes beta decay, resulting in a daughter nucleus with the same number of nucleons as the parent nucleus.

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- \quad (\textit{incomplete expression}) \qquad \ldots(4.1)$$

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+ + \quad (\textit{incomplete expression}) \qquad \ldots(4.2)$$

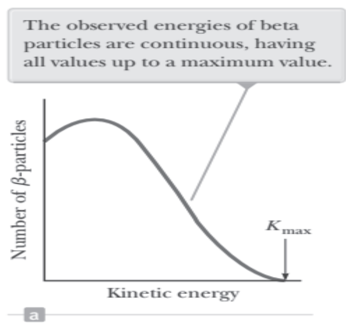
A beta particle can be either a positron (e^+) or an electron (e^-), with the latter being the more widespread name. *These formulations do not adequately represent beta decay*. We'll explain why this is the case in a moment.

As with alpha decay, beta decays preserve both the nucleon number and total charge. Due to the fact that A does not change but Z does, we conclude that beta decay occurs when either a neutron transforms into a proton (Eq. 4.1) or a proton transforms into a neutron (Eq. 4.2). Take note that the electron or positron emitted in these decays is not present in the nucleus prior to the decay; it is formed during the decay process from the decaying nucleus's rest energy. There are two distinct beta-decay processes.

$${}^{14}_{6}C \longrightarrow {}^{14}_{7}N + e^- \quad (\textit{incomplete expression}) \qquad \ldots(4.3)$$

$${}^{12}_{7}N \longrightarrow {}^{12}_{6}C + e^+ \quad (\textit{incomplete expression}) \qquad \ldots(4.4)$$

Let's take a look at the energy of the system experiencing beta decay before and after the decay is complete. The energy of the isolated system must be conserved in the same way as alpha decay. While alpha decay occurs at discrete energy, it is observed that beta particles from a single type of nucleus are emitted over a continuous range of energies (Fig. 4.1a) (Fig. 4.1b). *A drop in rest energy results in a decrease in kinetic energy, which is equal to the Q value.* The Q value must be the same for each decay since all nuclei in the sample have the same beginning mass. The variety of kinetic energies of the released particles is displayed in Figure 1a, so what is the reason for this? The law of conservation of energy and the isolated system model appears to have been broken. A closer look at Equations 4.1 and 4.2 shows that the rules of conservation of angular momentum (spin) and linear momentum are also violated.



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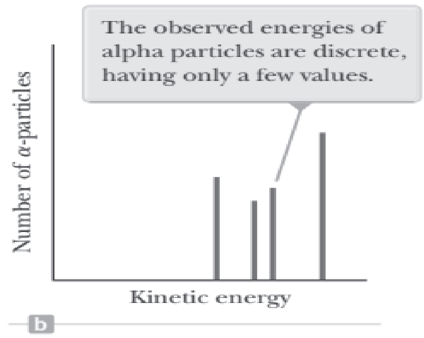

Fig 4.1: *(a) Beta-particle energies in a typical beta decay (b) Alpha-particle energies in a typical alpha decay*

Pauli postulated in 1930, following much experimental and theoretical research, that a third particle must be present in the decay products to transport away the "missing" energy and momentum. Fermi eventually dubbed this particle the **neutrino** (small neutral one) due to the particle's requirement to be electrically neutral and possess little or no mass. Although it escaped discovery for many years, the neutrino (symbol υ, Greek nu) was experimentally discovered in 1956 by Frederick Reines (1918–1998), for which he was awarded the 1995 Nobel Prize in Physics. The neutrino possesses the following characteristics:

Properties of the Neutrino

- It contains no electrical charge.
- Its mass is either zero (in which case it travels at the speed of light) or extremely less; compelling experimental evidence reveals that the neutrino's mass is not zero. Current experiments set the top limit on the neutrino's mass at about 7 eV/c^2.
- It has a spin of , which enables beta decay to satisfy the rule of conservation of angular momentum.
- It interacts with matter very weakly and is thus extremely difficult to detect.

Beta decay processes: Now we can express the beta-decay processes (Eqs. 4.1 and 4.2) correctly and completely:

$${}^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}Y + \beta^-(\equiv e^-) + \bar{\upsilon} + [\gamma] \quad \textbf{(\textit{complete expression})} \qquad \ldots(4.5)$$

$${}^{A}_{Z}X \longrightarrow {}_{Z-1}^{A}Y + \beta^+(\equiv e^+) + \upsilon + [\gamma] \quad \textbf{(\textit{complete expression})} \qquad \ldots(4.6)$$

As well as those for carbon-14 and nitrogen-12 (Eqs. 4.3 and 4.4):

$${}^{14}_{6}C \longrightarrow {}^{14}_{7}N + \beta^-(\equiv e^-) + \bar{\upsilon} + [\gamma] \quad \textbf{(\textit{complete expression})} \qquad \ldots(4.7)$$

$${}^{12}_{7}N \longrightarrow {}^{12}_{6}C + \beta^+(\equiv e^+) + \upsilon + [\gamma] \quad \textbf{(\textit{complete expression})} \qquad \ldots(4.8)$$

where the symbol $\bar{\upsilon}$ denotes the **antineutrino**, the neutrino's antiparticle. For the time being, suffice it to say that positron decay produces a neutrino and electron decay produces an antineutrino. The decays indicated above are evaluated using conservation laws, just as alpha decay, although relativistic equations must be employed for beta particles because their kinetic energy is large (usually 1 MeV) compared to their rest energy of 0.511 MeV. The decays indicated by Equations 4.7 and 4.8 are depicted graphically in Figure 4.2.

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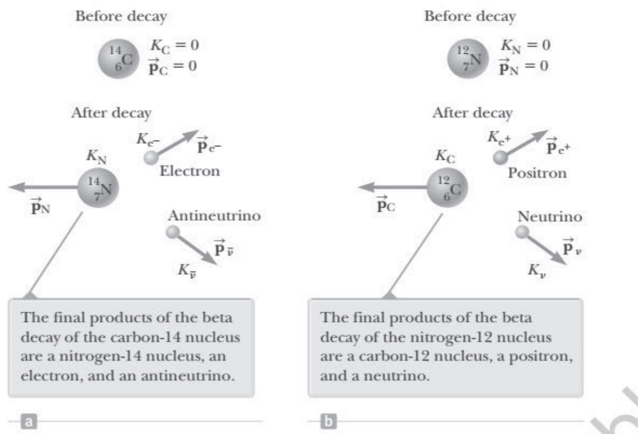

Fig 4.2 *(a) The beta decay of carbon-14 (b) The beta decay of nitrogen-12*

The number of protons in Equation 4.5 has grown by one while the number of neutrons has dropped by one. The fundamental process of decay can be expressed in terms of a neutron converting into a proton as follows:

$$n \longrightarrow p + + \bar{\upsilon}. \qquad \ldots(4.9)$$

The electron and antineutrino are ejected from the nucleus, resulting in an increase in the number of protons and a decrease in the number of neutrons, commensurate with the changes in Z and A - Z. In decay, a proton undergoes a similar transformation, transforming into a neutron, a positron, and a neutrino. This latter process can occur only within the nucleus, resulting in a decrease in nuclear mass. It is impossible for a solitary proton to experience this since its mass is less than that of a neutron.

The electron and the antineutrino are ejected from the nucleus, with the net result that there is one more proton and one fewer neutron, consistent with the changes in Z and A - Z. A similar process occurs in decay, with a proton changing into a neutron, a positron, and a neutrino. This latter process can only occur within the nucleus, with the result that the nuclear mass decreases. It cannot occur for an isolated proton because its mass is less than that of the neutron.

Electron Capture: It occurs when a parent nucleus catches one of its own orbital electrons and produces a neutrino, is a process that competes with decay. After decay, the end result is a nucleus with a charge of Z - 1:

$$^{A}_{Z}X + {}^{0}_{-1}e \longrightarrow {}_{Z-1}^{\;\;A}Y + \upsilon \qquad \ldots(4.10)$$

In the majority of situations, a K-shell electron is captured, and so the process is referred to as ***K capture***. A simple illustration is the capture of an electron by :

$$^{7}_{4}Be + {}^{0}_{-1}e \longrightarrow {}^{7}_{3}Li + \upsilon_{\upsilon}$$

In order to witness electron capture, the x-rays emitted by electrons cascading from higher-shell electrons to fill the void produced in the K shell are often used.

Furthermore, we define the Q values of beta-decay processes.

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$Q = (M_X - M_Y)c^2$

where M_X and M_Y are the neutral atom masses. The expression given above gives the Q values for decay and electron capture. The parent nucleus increases in atomic number during decay, and one electron must be absorbed by the atom in order for it to become neutral. The system contains a free electron both before and after the decay if the starting system is the neutral parent atom and an electron (which will eventually combine with the daughter to produce a neutral atom) and the final system is the neutral daughter atom and the beta-ejected electron. As a result, when the starting and end masses of the system are subtracted, the electron mass cancels.

Regarding decay, the Q values are given by,

$Q = (M_X - M_Y - 2m_e)c^2$

The atomic number of the parent lowers by one when the daughter is produced, necessitating the addition of $-2m_ec^2$. After the decay, the daughter atom loses an electron to become a neutral atom. As a result, the daughter atom, the ejected positron, and the shed electron are the end products.

In order to determine whether or not a procedure is feasible, these relationships can be used. It is possible that Q value for decay of a parent nucleus could be negative, as in the case of this parent nucleus. It does not happen in this scenario. In this case, the Q value for electron capture may be a positive number, therefore electron capture can occur even though decay is not conceivable. For example, the decay of illustrated above falls into this category.

Beta Spectrum

The energy released in a beta decay is released by three particles; the recoil nucleus, the beta electron, and its antineutrino. The nucleus, which is extremely heavy in comparison to the other two, consumes a minuscule fraction of the available energy, which is effectively split between the electron and antineutrino. On average, the electron carries little less than half (i.e., 50%) of this energy, while the antineutrino carries slightly more than half (i.e., 50%).

As in the case of electrons and antineutrinos, the electron and the neutrino share the electron's role in a beta-plus decay (which is extremely rare).

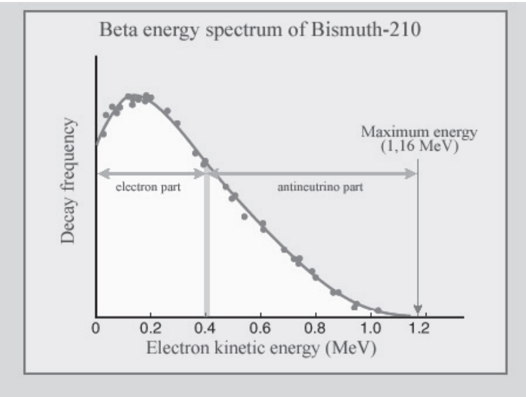

Fig 4.3 *An exchange of energy*

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The decay energy is shared among the nucleus, an electron, and an antineutrino in a beta decay like that of a bismuth-210 nucleus. Because the nucleus is so much more massive than the other two (its mass is 320 000 times that of the electron), it absorbs very little kinetic energy. As a result, the electron and the antineutrino share the energy. With the antineutrino eluding detection, only the beta electron with a fluctuating energy is visible. The picture depicts the bismuth-decay beta spectrum, which shows the distinctive energy distribution of beta electrons.

The beta electron energy distribution, often known as the beta spectrum, is distinctive because kinetic energy of the emitting nucleus is low, the electron and antineutrino share the decay energy in varying quantities. When an electron carries all of the decay energy, its energy is maximum. When it is the antineutrino, it becomes null.

Low-energy electrons dominate the beta spectrum's asymmetrical structure. In spite of its great lightness, the antineutrino carries more kinetic energy than the electron, which is heavier in comparison. Few beta electrons reach the maximum energy permitted, whereas most of them have low energies. This is in accordance with the rule.

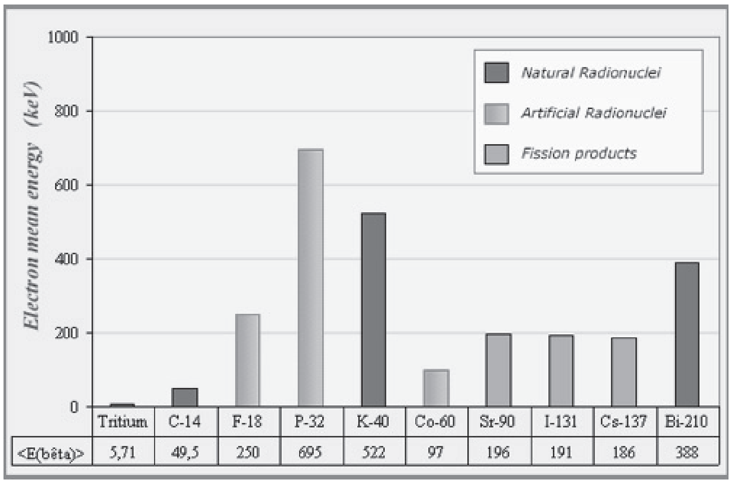

Fig 4.4 *Samples of Average Beta Energies*

Because no two radionuclides produce beta electrons with the same energy, the average beta energy is used to compare radionuclides. For example beta emitted by tritium has a hundred times less energy on average than beta emitted by phosphorus-32. Much lower than alpha particles, beta average energy are less than 1 MeV. (Usually above 4 MeV).

Since low-energy electrons are easier to stop, it is a good thing that beta electron have so much energy. When it comes to radiation safety, electrons' average energy is more important than their maximum energy. When compared to phosphorus-32, a powerful beta emitter with an energy of 695 keV, tritium, for example, has an average energy of 5.69 keV.

Beta electrons have lower energy than alpha particles, which always have energies above 4000 keV, whereas beta electrons have energies below 1 MeV in most situations (4 MeV). With the exception of potassium-40, the half-lives (or lifetimes) are substantially shorter.

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A beta decay is frequently associated with the emission of a gamma ray as a result of the nucleus being deexcited. This emission reduces the energy that the electron and antineutrino must share. For example, the available energy in the beta decay of caesium-137 is 1176 keV, but in 95% of cases, it is accompanied by the emission of a distinctive 662 keV gamma, lowering the available energy to 514 keV. The observed spectrum beta is the sum of the two spectra corresponding to the modes without or with gamma in proportions of 5% and 95%.

Additionally, it is possible for gamma to convey their energy to an atom's electron - this is known as internal conversion. These electrons are not exactly beta electrons and have their own distinct energy characteristics.

Check Your Progress

1. What happens when a radioactive nucleus undergoes beta decay?
2. When does electron capture occur?

4.3 NEUTRINO HYPOTHESIS

The secondary electrons giving rise to the discrete peaks are not emitted from the β-disintegrating nuclei. Only the electrons in the continuous part of the β-spectrum are emitted during β-disintegration of the nuclei.

Usually the areas under the discrete peaks are small compared to the area under the continuous distribution graph (not more than a few percent), which shows that the number of secondary electrons forming the peaks is only a few per cent of the total number of β-particles emitted.

Careful measurements have shown that the total number of electrons, including those in the peaks as well as in the continuous spectrum, is slightly greater than the number of nuclei undergoing β-decay. The latter is found to be equal to the number of electrons in the continuous spectrum, which shows that the electrons emitted during β-decay form the continuous spectrum only, excluding the peaks.

Both for β^- and β^+ decays, the emitted β-particles are found to have continuous distribution of energies or velocities ranging from 0 upto a maximum. In the case of electron capture, no observable particle is emitted from the nucleus. Only x-ray photons or Auger electrons, characteristic of the product atom, are observed.

We have seen before that during β-disintegration, the mass number A remains unchanged while the atomic number Z changes by one unit. This means that either a neutron is changed into a proton (as in β^- decay) or a proton is changed into a neutron (as in β^+ decay or in electron capture process) so that the total number of protons and neutrons ($Z + N = A$) remains unchanged.

Experimental study of α-disintegration shows that the α-particle spectra are discrete in nature, which points to the fact that the nuclei exist in discrete energy states, as expected from quantum mechanics for a closed micro-system. Transitions between these discrete levels in the parent and product nuclei give rise to the emission of mono-energetic groups of α-particles.

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The same conclusion is drawn from the study of the γ ray spectra. It may therefore be expected that due to the transitions between such discrete energy levels in the parent and product nuclei in the β-decay process, the β-particles will also be emitted with one or more definite energies, given by the energy difference between the initial and final states, less the mass energy of the β-particle. However, the observed continuous distribution of the β-energy is contrary to this expectation. Thus there is an apparent break down of the principle of conservation of energy in the case of β-decay. It may, however, be noted that the β-disintegration energy Q_β: agrees with the mass-energy difference between the parent and the product nuclei less the electron rest-energy for both β^- and β^+ disintegrations which also equals the maximum energy E of the emitted β-particles.

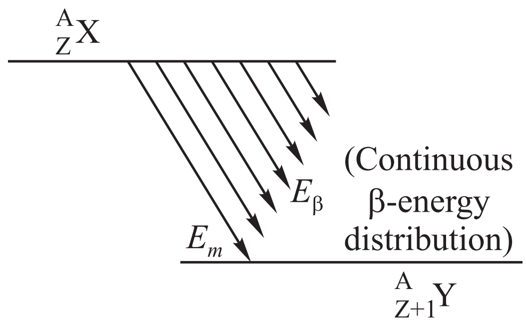

Fig. 4.5 *Beta transition between two discrete energy states of the parent (X) and product (Y) nuclei showing apparent breakdown of the conservation of energy.*

Another puzzling feature about the β-decay is the apparent failure of the principle of conservation of angular momentum. We know that the protons and the neutrons, constituting the nucleus of an atom, have intrinsic spin angular momentum $s = 1/2$ (in unit of $\hbar$) each. If the total number A of nucleons is even, then the total spin angular momentum of the nucleus $S = \Sigma s_i$ is either 0 or integral. On the other hand, if A is odd, then S will be half odd integral. In addition, the nucleons may have orbital angular momentum L which can only be integral multiples of $\hbar$. Since the total angular momentum of the nucleus (nuclear spin) is $I = L + S$, the value of I is integral or half odd integral in units of $\hbar$ depending on whether the number of nucleons A in the nucleus is even or odd respectively. For example, if A is even, I is integral or 0. So in the β-disintegration process, since A remains unchanged, I will remain integral or 0 *i.e.*, either I does not change or changes by an integral multiple of $\hbar$. Similar is the case when A is odd.

Now the electron has an intrinsic spin 1/2. So during its emission from the nucleus, it can carry away a half odd integral unit of angular momentum, since the orbital angular momentum change, if any, can take place by an integral multiple of $\hbar$. This means that the emission of an electron from a nucleus should change the angular momentum by a half odd integral multiple of $\hbar$ which however contradicts the statement made above that in the β-disintegration process I should change by an integral multiple of $\hbar$.

To explain these apparent inconsistencies, Wolfgang Pauli, in 1930 proposed that at the time of β-decay of a nucleus, a hitherto unobserved second particle, in addition to the electron, is emitted, which carries away the balance of energy $E_v = E_m - E_\beta$ so that the total energy of the two particles is equal to the maximum β-energy E_m. When the electron is emitted with zero kinetic energy, the second particle is emitted with the maximum energy $E_v = E_m$. On the other hand, when the electron is emitted with the energy E_m, the other particle has energy $E_v = 0$.

Here we have neglected the energy of the recoil nucleus undergoing β-decay, since it is much heavier than the particles emitted.

This new particle proposed by Pauli has been named the neutrino. It must have such physical properties that it would be very difficult to detect it. In fact it eluded observation for more than twenty five years after Pauli had proposed the neutrino hypothesis.

We can guess about some of the properties of the neutrino:

(*i*) The neutrino (ν) must be electrically neutral, so that the only change in the charge of the nucleus during β-decay is due to the emission of the electron or positron or due to the capture of an orbital electron. This is in agreement with observations.

(*ii*) The mass of the neutrino should be zero or very nearly so. This follows from the fact that the maximum energy E_m of the emitted electron is equal to the mass energy difference between the parent and the product nuclei less the rest energy of the electron. If the neutrino had a finite mass then its rest energy must also be subtracted to get E_m.

(*iii*) Intrinsic spin of the neutrino should be 1/2. Since the electron spin is also 1/2, two spin 1/2 particles are emitted during β-decay. Hence the two together will take away an integral unit of angular momentum, which is in agreement with the statement made above regarding the change of angular momentum in β-decay.

(*iv*) The neutrino must obey Fermi-Dirac statistics like the electron since its spin is 1/2.

Enrico Fermi, the famous Italian physicist, was the first to work out a successful theory of β-decay, based on the neutrino hypothesis (1934). According to Fermi, β-decay occurs due to the transformation of a neutron into a proton inside the nucleus with the emission of an electron and an anti-neutrino ($\bar{\nu}$) which is the anti-particle of the neutrino, just as the positron is an anti-particle of the electron:

$$n \rightarrow p + e^- + \bar{\nu} \quad ...(4.11)$$

Such decay is actually observed in the case of a free neutron which has a half-life of 10.6 min, when it is outside the nucleus.

The reverse transformation of a proton into a neutron by the emission of positron and a neutrino also occurs inside the nucleus in β^+ decay:

$$p \rightarrow n + e^+ + \nu \quad ...(4.12)$$

However, this transformation cannot occur in the case of a free proton, because energy-conservation cannot be satisfied in this case, the proton being lighter than a neutron.

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It may be noted that electrons, positrons, neutrinos and anti-neutrinos escaping from the nucleus in β-decay do not exist initially inside the nucleus. They are born at the time of β-decay, just as the photon is born at the time of radiative transition in an atom (or in a nucleus). This is unlike the case of α-decay of a nucleus, since the two protons and two neutrons forming the α-particle already exist inside the disintegrating nucleus.

Unlike α-decay, which takes place only in the heaviest nuclei, β-decay can occur in a wide range of nuclei, starting from the lightest at $A = 1$ (in neutron) up to some of the heaviest nuclei known in nature or artificially produced.

The energy liberated in β-decay process also varies over very wide range. For example, in the decay ${}^{3}H \rightarrow {}^{3}He + \beta^{-}$, the decay energy is only 0.02 MeV while in the decay ${}^{12}B \rightarrow {}^{12}C + \beta^{-}$ it is 13.4 MeV.

A brief sketch of Fermi's theory of allowed β-decay It gives a mathematical expression for the energy (or momentum), distribution of the β-particles.

The physical properties of the neutrinos are such that they are very difficult to detect. Since they carry no charge, they cannot produce ionization in matter. So the usual methods of detection of charged particles cannot be applied for their detection. Since they are practically massless, they cannot transfer energy to any other particle by elastic collision. Hence the method applicable in the case of detection of an electrically neutral particle like the neutron cannot be used in their case. There are reasons to believe that the sun emits a huge flux of neutrinos. For this reason, the earth is being incessantly bombarded by neutrinos. It has been estimated that about 10^{14} neutrinos pass through the human body every second. However, the probability of their interaction with the atoms in the human body is so small that not even one such collision takes place in a whole year. As stated before, there was no direct evidence for the detection of the neutrino for a long time. Finally in 1956, two American scientists, F. Reines and C.L. Cowan Jr., were successful in detecting the neutrino directly.

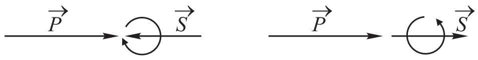
As stated before, the neutrino (ν) has an anti-particle, known as the anti-neutrino ($\bar{\nu}$). The former is emitted at the time of β^{+} decay and electron capture process, while the latter is emitted at the time of β^{-} decay. We can represent these processes symbolically as follows:

β^{-} decay: $$\underset{(l=0}{{}^{A}_{Z}X} \rightarrow \underset{0}{{}^{A}_{Z+1}Y} + \underset{1}{\beta^{-}} + \underset{1)}{\bar{\nu}} \quad ...(4.13)$$

β^{+} decay: $$\underset{(l=0}{{}^{A}_{Z}X} \rightarrow \underset{0}{{}^{A}_{Z-1}Y} + \underset{-1}{\beta^{-}} + \underset{1)}{\nu} \quad ...(4.14)$$

Electron capture: $$\underset{(l=0}{{}^{A}_{Z}X} + \underset{1}{e^{-}} \rightarrow {}^{A}_{Z-1}Y + \underset{1)}{\nu} \quad ...(4.15)$$

The neutrino and anti-neutrino are both mass-less and charge-less particles with the same intrinsic spin (1/2). It is believed that the difference between them lies in the fact that the spin vector *S* of the neutrino is anti parallel to its linear momentum p, while for the anti-neutrino the two vectors are parallel, as shown in Fig. 4.6.



(*a*) (*b*)

Fig. 4.6 *Neutrino and anti-neutrino.*

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The weakly interacting particles like the electron, positron, neutrino and antineutrino belong to a class of elementary particles called *leptons*. It is usual to associate a *lepton number l* with them. For the electron and the neutrino we put $l = +1$, for the antiparticles positron and anti neutrino, we put $l = -1$. At the time of β-decay, there is conservation of the lepton number, which means that l remains the same before and after the decay. As an example, in β^- decay $l = 0$ on the left side of Eq. (4.13). On the right side, the total lepton number is $1 - 1 = 0$. So the lepton number is conserved. Similarly for the β^+ decay. In the case of electron capture decay, $l = +1$ both on the left and right sides of Eq. (4.15). So lepton number conservation is satisfied.

4.4 FERMI THEORY OF BETA DECAY: ALLOWED AND FORBIDDEN TRANSITIONS PARITY VIOLATIONS IN BETA DECAY

Using Pauli's neutrino hypothesis in 1934, Fermi developed a successful theory of beta decay. Using Fermi theory, we may calculate the probability (or rate) of beta decay.

The theory is predicated on the following premises:

1. First and foremost, since the electron and neutrino don't exist before decay, the theory must explain how these particles are generated.
2. There must be a relativistic treatment of the electron and neutrino, as well.
3. The calculation must produce a uniform distribution of electron energies.
4. The interaction that generates the quasi-stationary states is weaker than the interaction that causes beta decay. Beta decay is caused by a relatively small amount of (time-dependent) nuclear potential, while stationary states are created by a much larger amount of (time-independent) nuclear potential. Since we can treat the interaction that causes beta decay as a weak disturbance in time-dependent perturbation theory, we can apply it to the process.

Suppose that the system's total Hamiltonian is

$$H = H_N + H^P(t)$$

$H^P(t)$ is smaller than nuclear potential H_N and is thought to be responsible for beta decay. In beta decay, we now know:

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$|\psi_i>$ ⟶ $|\psi_f>$

Initial state represents the state vector/ wave function of parent nucleus

Final state represents the combined state vector/wave function of duaghter nucleus and decay particles (beta particles and neutrinos)

The transition rate from beginning to final state, or decay probability, can then be calculated using the Fermi Golden rule:

$$\lambda = \frac{2\pi}{\hbar} |\langle \psi_f | \hat{H}^p | \psi_i \rangle|^2 \rho(E_f)$$

$$\lambda = \frac{2\pi}{\hbar} |H_{if}^p|^2 \rho(E_f) \quad \ldots(4.16)$$

where the matrix elements

$$H_{if}^p = \int \psi_f^* H^p \psi_i d\tau$$

with H^p representing the interaction potential responsible for beta decay. Also, note that here we use λ instead of W_{if} as both are same.

While ρ (E_f) is the density of final states, which can also be written as

$$\rho(E_f) = \frac{dn}{dE_f} \quad \ldots(4.17)$$

dn is the number of final states contained inside the energy interval dE_f. As previously stated, if there are a high number of accessible end states, a particular transition is more likely to occur.

As previously stated, beta decay

$\psi_i \equiv \psi_p$ (wave function of parent nucleus)

$\psi_f \equiv \psi_d \ \psi_e \ \psi_\upsilon$ (combined wave function of duaghter nucleus, beta-particle and anti-neutrino/neutrino)

The wave functions of electrons and neutrinos are referred to as ψ_e and ψ_υ, respectively. The wave function of a daughter nucleus is given by Ψ_d;

That's why matrix elements transform as follows:

$$H_{if}^p = \int \psi_d^* \psi_e^* \psi_\nu^* H^p \psi_i d\tau \quad \ldots(4.18)$$

Beta particles and neutrinos are now free to move around as they were before their formation. The related wave functions therefore have the standard free particle's wave function shape adjusted within the volume V. (which is nuclear volume for beta decay case).

$$\psi_e = \frac{1}{\sqrt{V}} e^{i \boldsymbol{k_e} \cdot \boldsymbol{r}}$$

Because the wave vector = , where is the electron's momentum, the electron wave function becomes;

$$\psi_e = \frac{1}{\sqrt{V}} e^{i\frac{\mathbf{p_e}\cdot\mathbf{r}}{\hbar}} \quad \text{...(4.19)}$$

In the same way, the wave function for neutrinos is:

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$$\psi_\nu = \frac{1}{\sqrt{V}} e^{i\frac{\mathbf{p_\nu}\cdot\mathbf{r}}{\hbar}} \quad \text{...(4.20)}$$

Typically, the kinetic energy of beta particle is 1 *MeV*. Then, the momentum is = 1.4 *MeV/c*.

With the momentum = 1.4 *MeV/c*, if we calculate

$$\frac{p_e}{\hbar} = \frac{1.4\,x\,1.6\,x\,10^{-13} J}{3\,x\,10^8 m/s} \frac{1}{1.054\,x\,10^{-34} J-s}$$

$$\frac{p_e}{\hbar} = \frac{0.708}{10^{-13} m}$$

$$\frac{p_e}{\hbar} \approx \frac{0.007}{10^{-15} m} = 0.007\,fm^{-1}$$

For a typical nuclear radius $r = 1\,fm$,

≈ *0.007*

Therefore,

$$<<1 \quad \text{...(4.21)}$$

Now, from equation (4.19), the electron wave function can be expanded as:

$$\psi_e = \frac{1}{\sqrt{V}} e^{i\frac{\mathbf{p_e}\cdot\mathbf{r}}{\hbar}}$$

$$\psi_e = \frac{1}{\sqrt{V}} \left(1 + i\frac{\mathbf{p_e}\cdot\mathbf{r}}{\hbar} + ...\right) \quad \text{...(4.22)}$$

Using the condition <<1, we can keep only first term and all the higher order terms can be neglected. As a result,

$$\psi_e \approx \frac{1}{\sqrt{V}} \quad \text{...(4.23)}$$

This approximation is known as the **allowed approximation**.

Neutrino's wave function can be approximated by ignoring higher order terms in its exponential. A new wavefunction for neutrinos is written as follows:

$$\psi_\nu \approx \frac{1}{\sqrt{V}} \quad \text{...(4.24)}$$

The matrix element can now be calculated using the modified wavefunction of electron and neutrino stated in equations (4.23) and (4.24):

$$H_{if}^p = \int \psi_d^* \psi_e^* \psi_\nu^* H^p \psi_i \, d\tau$$

NOTES

$$H_{if}^{p}=\int \psi_d^{*}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{V}}H^{p}\psi_i d\tau$$

$$H_{if}^{p}=\frac{1}{V}\int \psi_d^{*}H^{p}\psi_i d\tau$$

$$H_{if}^{p}=\frac{1}{V}M_{if} \qquad \ldots(4.25)$$

where,

$$M_{if}=\int \psi_d^{*}H^{p}\psi_i d\tau$$

is referred to as **nuclear matrix elements** since it expresses just the waves of the parent and daughter nuclei.

The revised formulation of transition rate is obtained by substituting the expression of from equation (4.25) into equation (4.16).

$$\lambda=\frac{2\pi}{\hbar}|H_{if}^{p}|^{2}\rho(E_f)$$

$$\lambda=\frac{2\pi}{\hbar}\frac{1}{V^{2}}|M_{if}|^{2}\frac{dn}{dE_f} \qquad \ldots(4.26)$$

For the time being, we can treat the nuclear matrix element M_{if} as a constant quantity while computing the density of states $\rho(E_f) = dn/dE_f$ in order to determine the transition probability/rate. To put it another way, the final state density dictates the probability of a transition. Because of this, the **beta energy spectrum** is determined by the density of states.

A final quantum state can be assigned to the daughter nucleus, therefore it is important to keep this in mind. As free particles, the decay products (electron and neutrino) might have continuum energy states in contrast to this. For this reason, we need to know how many final states are accessible to the decay products before we can calculate the density of states.

Let us assume that an electron (or positron) is emitted with a momentum p_e in order to obtain the number of electron quantum states. We have no interest in the current momentum. Using this method, one can approximate the states for the range from p to $p+dp$ by using the following:

Due to the fact that the electron is supposed to be free, its position and momentum can be described with uncertainty dx, dy, dz, dp_x, dp_y, dp_z.

$$dx\, dp_x \sim h$$
$$dy\, dp_y \sim h$$
$$dz\, dp_z \sim h$$

Then, in quantum physics, the smallest volume in phase that can be measured is:

$$dx\, dp_x\ dy\ dp_y\, dz\ dp_z \sim h^3$$

Nevertheless, we may say that this volume corresponds to the particle's single quantum state, as we cannot know the particle's position or momentum within the volume.

NOTES

Since the total number of quantum states in a given volume V and momentum range p to $p+dp$ can only be calculated by integration, we must now do it for the corresponding phase space volume.

$$V_{phase} = \int dx\,dy\,dz \int_{p}^{p+dp} dp_x\,dp_y\,dp_z$$

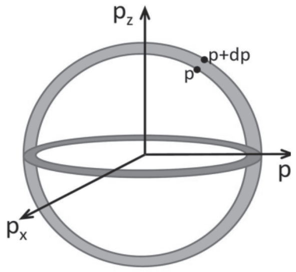

Fig 4.7 $V_{phase} = V\,4\pi\,p^2\,dp$

The number of quantum states corresponding to a V phase space volume is equal to the h^3 phase volume of one quantum state.

$$dn_e = \frac{4\pi\,p^2\,dp\,V}{h^3}$$

Essentially, dn_e is the number of quantum states possible to an electron contained in a spatial volume V with a velocity ranging from p to $p+dp$.

Analogously, we can determine the number of quantum states possible to a neutrino confined in a spatial volume V with a momentum ranging from q to $q+dq$ and denoted by:

$$dn_\nu = \frac{4\pi\,q^2\,dq\,V}{h^3}$$

Then the total number of final states which have simultaneously an electron and a neutrino (confined in spatial volume V) with momenta p to $p+dp$ and q to $q+dp$ are:

$$dn = dn_e\,dn_\nu$$

$$dn = \frac{(4\pi)^2 V^2 p^2\,dp\,q^2\,dq}{h^6} \qquad \ldots(4.27)$$

If we use this expression of dn in equation (4.26), the expression for the transition probability modifies as:

$$\lambda = \frac{2\pi}{\hbar}\frac{1}{V^2}|M_{if}|^2\frac{dn}{dE_f}$$

$$\lambda = \frac{2\pi}{\hbar}|M_{if}|^2(4\pi)^2\frac{p^2\,dp\,q^2}{h^6}\frac{dq}{dE_f}$$

NOTES

4.5 CONCEPT OF HELICITY

Electron and anti neutrino are longitudinally polarized which means that their spins are aligned parallel (for $\overline{\nu}$) or antiparallel (for β^-) to their respective linear momenta for backward emission of the β^- particle.

Though the polarization of the β-particles observed in β-decay is not a general characteristic of the electrons, it is regarded as a basic property of the antineutrino (and of the neutrino) and is known as *helicity* (*H*).

We define helicity as

$$H = \frac{\sigma \cdot p}{|\sigma|\,|p|} \qquad ...(4.28)$$

where σ is the spin of the neutrino and *p* is its momentum. The Pakistani physicist Abdus Salam and L. Landau of Russia were the first to point out, on the basis of the two component theory of the neutrino (assuming the neutrino mass $m_v = 0$), that helicity should be, a fundamental property of the neutrino. To understand this we note that for a particle with a finite mass, the velocity can be different in different frames of reference. If, for instance, the particle spin is parallel to its momentum in some frame of reference, then to an observer moving faster than it in the previous frame, it will appear to be moving in the opposite direction and hence has momentum opposite to its spin. So it will have different polarizations in the different frames of reference. However, this is not the case for a massless particle (fermion) which must always move with the velocity of light *c*. Hence there can be no frame of reference which will move faster than it.

From the expression given above, the helicity is $H = \pm 1$ depending on whether the relative orientation of the spin and momentum of the particle is parallel or antiparallel. According to the two component theory mentioned above, the neutrino has $H = -1$ (anti parallel orientation of spin relative to the momentum) while the antineutrino has $H = +1$ (parallel orientation of spin relative to momentum). This is the only distinction between the two particles.

If spin is regarded as a rotation, then the motion of the neutrino is analogous to that of a *left handed screw* while the motion of the antineutrino is similar to the motion of a *right handed screw*.

The existence of a definite helicity of the neutrino is directly related to the violation of parity conservation in weak interaction. If a particle has right-left symmetry, then upon mirror reflection, the wave-function either remains the same or simply changes sign, while the particle is transformed to itself. However, a particle with a definite helicity does not possess right-left symmetry. So upon mirror reflection, a right handed screw-like particle transforms into a left handed screw-like particle. Thus the particle is not transformed to itself, which means violation of parity conservation.

Measurement of neutrino helicity:

The helicity of neutrino was measured directly in an experiment performed by M. Goldhaber, L. Grodzins and A.W. Sunyar (1958). They used as source the K-capturing ^{152}Eu (τ = 9.3 h) isomer which has the decay scheme.

NOTES

The product nucleus ^{152}Sm goes to the ground state by γ-emission (E_γ = 961 keV) which is an EI transition. In the experiment, resonance fluorescence produced by the γ-rays were studied. Because of the recoil of the emitting nucleus ^{152}Sm, the γ-energy was reduced by about E_r = 3.26 eV from the transition energy. Because of the very short half-life of the excited state, its width Γ is relatively large, being about 0.02 eV. Even so, it is not wide enough to compensate for the recoil energy-loss ($\Gamma << E_r$). However, the compensation is provided by Doppler shift due to the recoil velocity of the source, which is a product in the K-capture decay of the parent nucleus ^{152}Eu emitting a neutrino of energy E_v = 900 keV. This recoil energy is about 2.86 eV. Notice that there are two different types of recoil of the ^{152}Sm nucleus due to two different reasons: the first due to neutrino emission in the electron capture by the parent nucleus and the second due to γ-emission from the excited product nucleus ^{152}Sm*.

Since the compensation due to Doppler shift is slightly less than the recoil energy change of the γ-rays, the γ-rays were allowed to proceed at an angle slightly less than 180° w.r.t. the direction of emission of the neutrino in K-capture decay of ^{152}Eu, as can be seen from Fig. 4.8(*a*), showing the experimental arrangement of Goldhaber et al. The theme of the experiment is illustrated in Fig. 4.9.

We can write down the law of conservation of angular momentum in the two successive transitions involved as below:

$$\underset{0}{^{152}\text{Eu}^{m}} + \underset{\frac{1}{2}(\uparrow)}{e_K^-} \rightarrow \underset{1(\uparrow)}{^{152}\text{Sm}^{*}} + \underset{\frac{1}{2}(\downarrow)}{\nu}$$

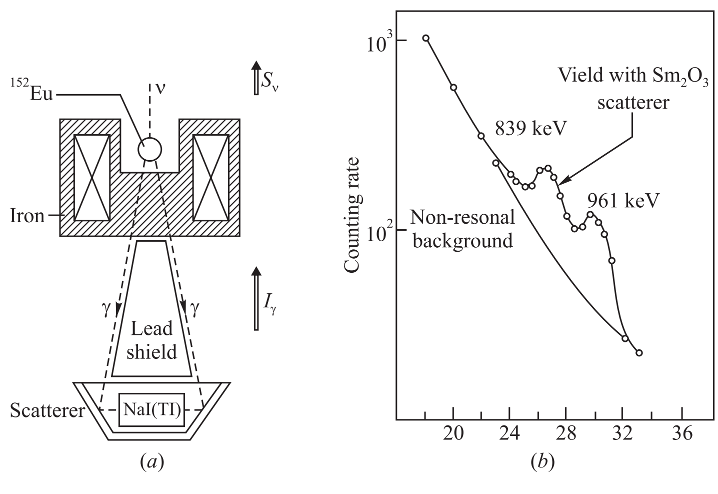

Fig. 4.8 *(a) Experiment of Goldhaber and others on the measurement of the helicity of the neutrino. (b) Results of the neutrino helicity experiment.*

In this case angular momentum will be conserved if the spins of the neutrino and the ^{152}Sm* nucleus are oriented oppositely. Since their momenta are also in opposite directions, it follows that the longitudinal polarization of the nucleus ^{152}Sm* must have the same sign as that of helicity of the neutrino. In the second transition we have

$$\underset{1}{^{152}\text{Sm}^{*}} \rightarrow \underset{0}{^{152}\text{Sm}} + \underset{1}{\gamma}$$

NOTES

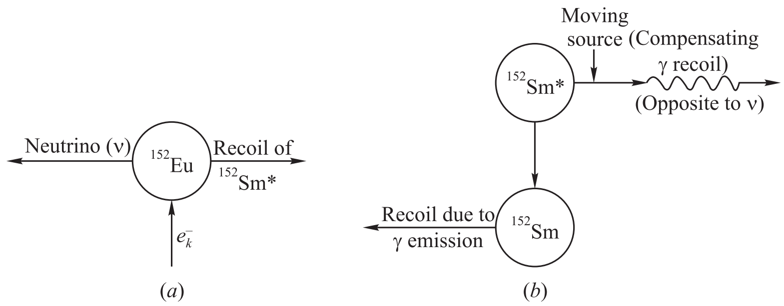

Fig. 4.9 *Theme of the experiment on the measurement of neutrino helicity.*

The magnetized iron used as a polarimeter [see Fig. 4.8 (*a*)] allows only those γ-rays to be transmitted which have circular polarization such that their spins are up and hence the spin of the 1^- state of ^{152}Sm must also be up. Thus the sign of the circular polarization must be the same as the sign of the longitudinal ploarization of the emitting nucleus and hence it is the same as the sign of the neutrino helicity.

So the experiment boils down to the measurement of the sign of the circular polarization of the γ-rays. This is measured from the change in the number ot counts in the γ-ray detector upon a reorientation of the magnetic field in the magnetized iron. The results are shown in Fig. 4.8(*b*).

The helicity of the neutrino was found to be negative. Though there is no direct experimental determination of the helicity of the anti-neutrino, all other experimental data show that it must be positive.

It was seen above that the electrons emitted in β-decay are longitudinally polarized (see Wu's experiment). The longitudinal polarization of the β-rays have been measured and is given by

$$P(\beta^{\pm}) = \pm\frac{v}{c} \qquad ...(4.29)$$

Each lepton has a lepton number (also called the leptonic *charge*). For electrons and ve the leptonic charge is +1 while for positrons and $\overline{\nu}_e$ it is –1. We conclude that the sign of longitudinal polarization of the electronic leptons is opposite to the sign of their leptonic charge.

4.6 MULTIPOLE TRANSITION AND SELECTION RULES FOR THE DECAY OF THE NUCLEI

Assuming an electric dipole interaction between a nucleus and e.m. field, we found a transition rate that was above the rate at which an electric dipole radiation is emitted. Because of this sort of radiation, only one quantum of angular momentum may be carried out of the nucleus (i.e., $\Delta l = \pm 1$, between excited and ground state). As a general rule, excited levels are more than one l *l* apart, and so the radiation that is emitted must be higher multipole.

1. **Electric Multipole:** To return to the expansion of the radiation interaction in multipoles, consider the following:

$$\hat{V} \sim \sum_{l} \frac{1}{l!}(i\hat{\vec{k}} \cdot \hat{\vec{r}})^l$$

where is interaction potential.

NOTES

Then the transition rate becomes:

$$\lambda(El) = \frac{8\pi(l+1)}{l[(2l+1)!!]^2}\frac{e^2}{\hbar c}\left(\frac{E}{\hbar c}\right)^{2l+1}\left(\frac{3}{l+3}\right)^2 c\left\langle|\hat{\vec{r}}|\right\rangle^{2l}$$

Notice the strong dependence on the *l* quantum number. Setting again

$$|\left\langle\hat{\vec{r}}\right\rangle| \sim r_0 A^{1/3}$$

we also have a strong dependence on the mass number.

Thus, we have the following estimates for the rates of different electric multipoles:

- $\lambda(E1) = 1.0 \times 10^{14} A^{2/3} E^3$
- $\lambda(E2) = 7.3 \times 10^7 A^{4/3} E^5$
- $\lambda(E3) = 34 A^2 E^7$
- $\lambda(E4) = 1.1 \times 10^{-5} A^{8/3} E^9$

2. **Magnetic Multipoles:** The e.m. potential can also contain magnetic interactions, leading to magnetic transitions. The transition rates can be calculated from a similar formula:

$$\lambda(Ml) = \frac{8\pi(l+1)}{l[(2l+1)!!]^2}\frac{e^2}{\hbar c}\frac{E}{\hbar c}^{2l+1}\left(\frac{3}{l+3}\right)^2 c\left\langle|\hat{\vec{r}}|\right\rangle^{2l-2}\left[\frac{\hbar}{m_p c}\left(\mu_p - \frac{1}{l+1}\right)\right]$$

where μ_p is the magnetic moment of the proton (and mp its mass). Estimates for the transition rates can be found by setting :

- $\lambda(M1) = 5.6 \times 10^{13} E^3$
- $\lambda(M2) = 3.5 \times 10^7 A^{2/3} E^5$
- $\lambda(M3) = 16 A^{4/3} E^7$
- $\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9$

4.7 INTERNAL CONVERSION, CONVERSION COEFFICIENTS OF ISOMERIC NUCLEI

Internal conversion is a mechanism of nucleus deexcitation that competes with gamma emission for energy. It occurs when the nucleus is in an excited state as a result of beta or alpha radioactive decay. Internal emission can be compared to gamma emission, in which the gamma fades when it interacts with one of the atom's atomic electrons. As a result, it is often referred to as electronic conversion.

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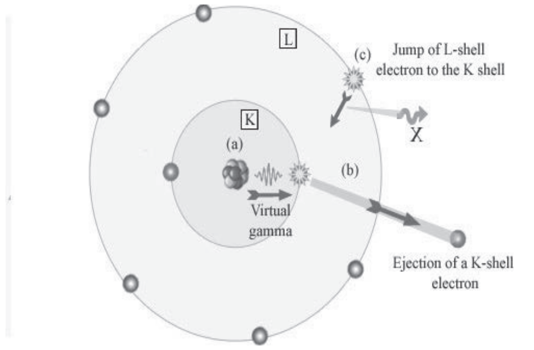

Fig 4.10 *A gamma ray is emitted by an excited nucleus (a). One of the electrons in the innermost layer of the atom (b) is most likely to be hit by this gamma ray. The atom loses an electron. The gamma gets absorbed by the electron and disappears. Because of this, the electron is no longer present in that layer. Reorganization of the atom takes place, with one electron from layer L filling up the vacuum (c). It generates an X-ray.*

The gamma energy is passed on to the expelled electron, but it must break free of the atom's gravitational pull. It loses some of its atomic binding energy once it is liberated. Remembering the atom's shell structure, the electron's binding energy is the energy of the layer to which it belongs. Energy from the gamma reduced electron's unique binding energy on its atomic layer is transmitted to its electrons. It is clear that both the gamma and the binding energy have well-established values. Since the electron is being expelled, it will have a range of energy levels (one for each layer). For electrons in the innermost K layer, the internal conversion probability is the highest, and declines rapidly with the outermost layers.

Conversion electrons have a fixed energy, in contrast to beta decay electrons, whose energy varies between 0 and a maximum value, with a portion of the decay energy carried by an invisible neutrino.
Following the electron expulsions, the electron atomic cloud undergoes re-arrangement, resulting in the emission of X-rays.

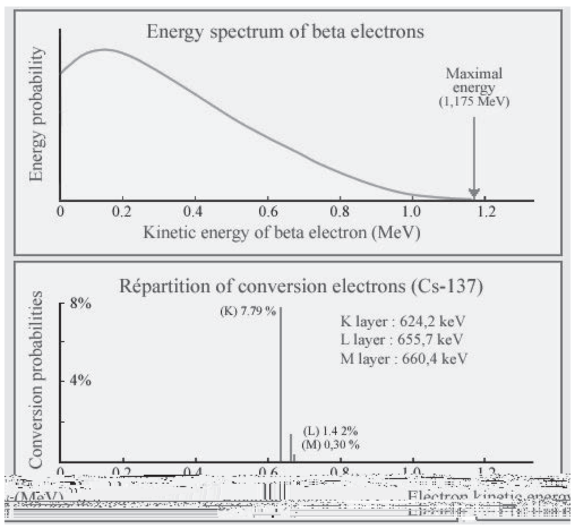

Fig 4.11 *Beta electrons and conversion electrons; Caesium-137, a well-known beta-ray emitter, is the example chosen. In 85.1 %, the beta electron is accompanied by a 661.57 keV gamma ray, while in 9.6 %, an electron conversion occurs. While the energy distribution of beta electrons is constant, the energy of conversion electrons varies depending on which atomic layer they originate from. Layer K is where the most conversions happen. Conversion electrons carry around 5% of the decay energy in the case of caesium-137.*

The weights of gamma emission and internal conversion are shown in the example of cesium-137. 94.7 % of beta decays result in a nucleus in an excited state, with 85.1 % returning to the ground state by emitting an energetic 661.57 keV gamma and 9.6 % returning to stability through internal conversion. The electron conversion energy is slightly smaller than the gamma conversion energy.

In general, the existence and energy of conversion electrons stay related to the nucleus's gamma rays. Their contribution to the decay energy, which is added to the energy of beta electrons, is only a few percent at most.

Internal Conversion Coefficient (α)

The internal conversion coefficient is a term used in nuclear physics to characterize the rate of internal conversion.

The internal conversion coefficient may be empirically determined by the following formula:

$$\alpha = \frac{number\ of\ de-excitations\ via\ electron\ emission}{number\ of\ de-excitations\ via\ \gamma-ray\ emission}$$

For E_0 (electric monopole) nuclear transitions, there is no appropriate formulation for an equivalent concept.

Internal conversion coefficients can be determined theoretically. Their correctness is largely accepted, however because the quantum mechanical models on which they are based only consider electromagnetic interactions between the nucleus and electrons, unexpected effects may occur, resulting in a conversion coefficient that differs from the one measured empirically.

Internal conversion coefficients can be found in tables, although this is tedious. Software solutions have been created to quickly and conveniently display internal conversion coefficients.

4.8 ANGULAR CORRELATION OF SUCCESSIVE DECAY

A directional connection exists between two successive gamma rays, which is unique due to the multipolarity of the transitions involved in the cascade. Experimental measurements of these correlations can provide information on the spins of the nuclear states involved.

Each nuclear state with angular momentum *J* contains a set of *m*-states,

$$m_J = -J, -J+1, -J+2, \ldots. J-2, J-1, J \tag{4.30}$$

They exhibit degenerate energies. The transition from one excited nuclear state to another is essentially a transition between various pairs of *m*-states, as seen in Fig. 4.12 for a dipole transition. There are three conceivable *m*-state transitions for a dipole transition. Each of these emits an angular distribution that is uniquely anisotropic. The observed angular distribution of the radiation emitted by $m_i \rightarrow m_f$ is;

$$\underline{W}(\theta) = \sum_{m_i} p(m_i)\, W_{m_i \rightarrow m_f}(\theta), \tag{4.31}$$

The specific angular distribution $W_{mi\rightarrow mf}(\theta)$ for emitted radiation between pairs of *m*-states in Fig. 4.12.

NOTES

Table 4.1

$m_i \rightarrow m_f$	$W_{m_i\rightarrow m_f}(\theta)$
$-1 \rightarrow 0$	$\frac{1}{2}(1+\cos^2\theta)$
$0 \rightarrow 0$	$\sin^2\theta$
$1 \rightarrow 0$	$\frac{1}{2}(1+\cos^2\theta)$

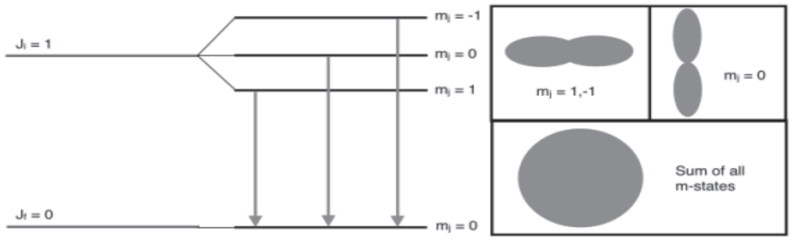

Fig 4.12 *A diagram illustrating how the decay $J_i \rightarrow J_f$ is actually caused by decays between m-states. Each transition between pairs of m-states emits radiation with an anisotropic angular distribution, but if all initial m-states are evenly populated, only an isotropic radiation distribution is observed.*

where, $p(m_i)$ denotes the starting state's population. The specific angular distributions $W_{mi\rightarrow mf}(\theta)$ for each transition between pairs of m-states are reported in Table 4.1. If each initial state (m_i) is equally occupied, and $p(m_{-1}) = p(m_0) = p(m_1) = \frac{1}{2}$, then the observed angular distribution is as follows.

$$W(\theta) \propto \frac{1}{3}[\frac{1}{2}(1+cos^2\theta)] + \frac{1}{3}(sin^2\theta) + \frac{1}{3}[\frac{1}{2}(1+cos^2\theta)], \quad (4.32)$$

W is constant at all angles θ; as gamma rays are isotropically emitted, no anisotropic angular distribution will be noticed.

To observe an anisotropic distribution of radiation, an unequal population in the initial *m*-states must be created. This can be accomplished by detecting the preceding radiation that populates those first *m*-states with such rarity. Continuing with the dipole radiation scenario from Figure 4.12, suppose that J_i was filled from another excited state with spin $J_0 = 0$, resulting in a $0 \rightarrow 1 \rightarrow 0$ cascade of gamma rays γ_1 and γ_2. (as shown in Fig. 4.13). If γ_1 is observed in a detector at a given position, a z-axis can be defined along its emission axis, so that γ_2 is observed at an angle of θ_2 to that axis (shown in Fig. 4.13). Between definition, the angle formed by γ_1 (θ_1) and the z-axis is 0. When $W_{m0\rightarrow m0}(\theta_1) = \sin^2\theta_1 = 0$, which indicates that m_j=0 cannot be populated. Now, an unequal population of *m*-states has been established that corresponds to J_i. The measured anisotropic angular distribution has the following shape:

$$W(\theta_2) \propto \frac{1}{2}[\frac{1}{2}(1+cos^2\theta_2)] + 0(sin^2\theta_2) + \frac{1}{2}[\frac{1}{2}(1+cos^2\theta_2)] \propto 1+cos^2\theta_2, \quad (4.33)$$

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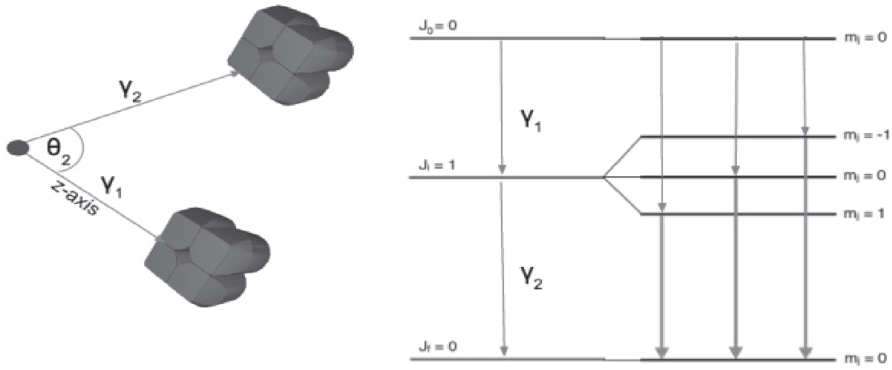

Fig 4.13 *(Left) A measurement of the angular correlation of a 0 → 1 → 0 cascade of successive gamma rays γ_1 and γ_2. The z-axis is defined as the direction of emission of γ_1, and θ_2 is the direction of detection of γ_2 with respect to the z-axis. (Right) A simplified level diagram illustrating the various m-states involved in the decay of the 0 → 1 → 0 cascade.*

which is not constant at all angles (plotted in Fig. 4.14).

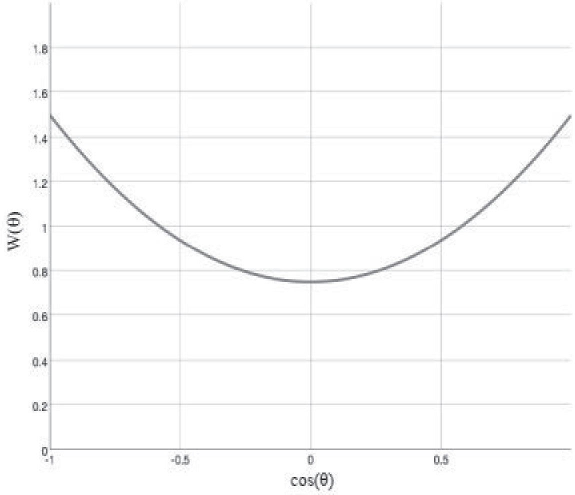

Fig 4.14 *A plot of the angular correlation expected to be observed for a 0 → 1 → 0 gamma ray cascade.*

In general, the angular correlation between two successive gamma rays can be written as

$$W(\theta) = 1 + A_2 P_2(cos\theta) + A_4 P_4(cos\theta), \quad (4.34)$$

where P_2 and P_4 are Legendre polynomials and A_2 and A_4 are spins of the nuclear states involved in the cascade, as well as the angular momenta and mixing ratios δ of each gamma ray. Each form of gamma ray cascade (0 → 1 → 0, 0 → 2 → 0, etc.) has a distinct angular correlation with distinctive A_2 and A_4 coefficients. Table 4.2 contains examples of selected cascades. Spin assignments for the nuclear states participating in a specific cascade can be determined by comparing theoretically predicted A_2 and A_4 coefficients to fits to experimental data.

NOTES

Table 4.2 *According to Eqn. 4.34, the theoretical A_2 and A_4 coefficients were employed to define the angular correlation of each mentioned cascade.*

Cascade	A_2	A_4
$0 \to 1 \to 0$	0.5	0
$0 \to 2 \to 0$	0.36	1.14
$1 \to 2 \to 0$	-0.25	0
$2 \to 2 \to 0$	0.25	0
$3 \to 2 \to 0$	-0.071	0
$4 \to 2 \to 0$	0.10	0.0091

Check Your Progress

3. Define helicity.
4. What is internal conversion?

4.9 ANSWERS TO 'CHECK YOUR PROGRESS'

1. A radioactive nucleus's atomic number changes by one when it undergoes beta decay, resulting in a daughter nucleus with the same number of nucleons as the parent nucleus.
2. Electron capture occurs when a parent nucleus catches one of its own orbital electrons and produces a neutrino, is a process that competes with decay.
3. The polarization of the b-particles observed in b-decay is not a general characteristic of the electrons, it is regarded as a basic property of the antineutrino (and of the neutrino) and is known as *helicity* (*H*).
4. Internal conversion is a mechanism of nucleus deexcitation that competes with gamma emission for energy.

4.10 SUMMARY

- A radioactive nucleus's atomic number changes by one when it undergoes beta decay, resulting in a daughter nucleus with the same number of nucleons as the parent nucleus.
- A beta particle can be either a positron () or an electron (), with the latter being the more widespread name.
- As with alpha decay, beta decays preserve both the nucleon number and total charge. Due to the fact that A does not change but Z does, we conclude that beta decay occurs when either a neutron transforms into a proton or a proton transforms into a neutron.
- In order to witness electron capture, the x-rays emitted by electrons cascading from higher-shell electrons to fill the void produced in the K shell are often used.
- The decay energy is shared among the nucleus, an electron, and an antineutrino in a beta decay like that of a bismuth-210 nucleus.
- Low-energy electrons dominate the beta spectrum's asymmetrical

structure. In spite of its great lightness, the antineutrino carries more kinetic energy than the electron, which is heavier in comparison.

- Beta electrons have lower energy than alpha particles, which always have energies above 4000 keV, whereas beta electrons have energies below 1 MeV in most situations (4 MeV).
- The secondary electrons giving rise to the discrete peaks are not emitted from the b-disintegrating nuclei. Only the electrons in the continuous part of the b-spectrum are emitted during b-disintegration of the nuclei.
- Both for b– and b+ decays, the emitted b-particles are found to have continuous distribution of energies or velocities ranging from 0 upto a maximum. In the case of electron capture, no observable particle is emitted from the nucleus.
- Another puzzling feature about the b-decay is the apparent failure of the principle of conservation of angular momentum.
- When the electron is emitted with zero kinetic energy, the second particle is emitted with the maximum energy $Ev = Em$. On the other hand, when the electron is emitted with the energy Em, the other particle has energy $Ev = 0$.
- It may be noted that electrons, positrons, neutrinos and anti-neutrinos escaping from the nucleus in b-decay do not exist initially inside the nucleus. They are born at the time of b-decay, just as the photon is born at the time of radiative transition in an atom (or in a nucleus).
- The physical properties of the neutrinos are such that they are very difficult to detect. Since they carry no charge, they cannot produce ionization in matter.
- The neutrino and anti-neutrino are both mass-less and charge-less particles with the same intrinsic spin (1/2).
- Using Pauli's neutrino hypothesis in 1934, Fermi developed a successful theory of beta decay. Using Fermi theory, we may calculate the probability (or rate) of beta decay.
- If spin is regarded as a rotation, then the motion of the neutrino is analogous to that of a *left handed screw* while the motion of the antineutrino is similar to the motion of a *right handed screw*.
- The helicity of neutrino was measured directly in an experiment performed by M. Goldhaber, L. Grodzins and A.W. Sunyar (1958). They used as source the K-capturing 152Eu (t = 9.3 h) isomer which has the decay scheme.
- Since the compensation due to Doppler shift is slightly less than the recoil energy change of the g-rays, the g-rays were allowed to proceed at an angle slightly less than 180° w.r.t. the direction of emission of the neutrino in K-capture decay of ^{152}Eu.
- Internal conversion is a mechanism of nucleus deexcitation that competes with gamma emission for energy. It occurs when the nucleus is in an excited state as a result of beta or alpha radioactive decay.
- The internal conversion coefficient is a term used in nuclear physics to characterize the rate of internal conversion.

- A directional connection exists between two successive gamma rays, which is unique due to the multipolarity of the transitions involved in the cascade. Experimental measurements of these correlations can provide information on the spins of the nuclear states involved.

NOTES

4.11 KEY TERMS

- **Nuclear decay:** It is the process by which an unstable atomic nucleus loses energy by radiation. A material containing unstable nuclei is considered radioactive.
- **Neutrino:** A neutrino is a subatomic particle that is very similar to an electron, but has no electrical charge and a very small mass, which might even be zero.
- **Doppler shift:** The Doppler effect or Doppler shift (or simply Doppler, when in context) is the change in frequency of a wave in relation to an observer who is moving relative to the wave source.
- **Helicity:** In physics, helicity is the projection of the spin onto the direction of momentum.

4.12 SELF ASSESSMENT QUESTIONS AND EXERCISES

Short Answer Questions

1. State the properties of neutrino.
2. How is the measurement of neutrino helicity done?
3. What do you understand by multipole transition?

Long Answer Questions

1. What do you understand by beta spectrum? Explain.
2. Describe the Fermi theory of beta decay.
3. Discuss angular correlation of successive decay.

4.13 FURTHER READING

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UNIT 5 ELEMENTS OF PARTICLE PHYSICS

NOTES

Structure

5.0 Introduction
5.1 Objectives
5.2 Classification of Elementary Particles and their Interactions
5.3 Concept of Quantum Numbers
5.4 Isospin Hypercharge
5.5 Strangeness
5.6 Lepton and Baryon Numbers
5.7 Invariance
5.8 Concept of Variation Laws and Selection Rules in Relation to Particle Production
5.9 Decay of Charge Conjugation
5.1 Parity Invariance and Time Reversal with Simple Application in Particle Physics
5.11 Elementary Idea Su(2) and Su(3) Results of Group Theory
5.12 Gell-Mann-Okubo Mass Formula (Without Derivation) and Its Application to Mass Spectra of Particles
5.13 Qualitative Idea of Quark Lepton Family and Quantum Chromodynamics
5.14 Answers to 'Check Your Progress'
5.15 Summary
5.16 Key Terms
5.17 Self Assessment Questions and Exercises
5.18 Further Reading

5.0 INTRODUCTION

Particle physics (also known as high energy physics) is a branch of physics that studies the nature of the particles that constitute matter and radiation. Particle physics usually investigates the irreducibly smallest detectable particles and the fundamental interactions necessary to explain their behaviour. Modern particle physics research is focused on subatomic particles, including atomic constituents, such as electrons, protons, and neutrons (protons and neutrons are composite particles called baryons, made of quarks), that are produced by radioactive and scattering processes; such particles are photons, neutrinos, and muons, as well as a wide range of exotic particles. In this unit, we will study in detail about the elementary particles, their classification and interactions, quantum numbers, invariance, decay of charge conjugation, elementary idea of SU(2) and SU(3), Gell-Mann-Okubo mas formula, quark model and quantum chromodynamics.

5.1 OBJECTIVES

After going through this unit, you will be able to:

- Discuss the classification of elementary particles and their interactions

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- Describe the concept of quantum numbers
- Explain the concept of variation laws and selection rules in relation to particle production
- Discuss the decay of charge conjugation
- Analyse the elementary idea of SU(2) and SU(3)
- State the Gell-Mann-Okubo mas formula along with its application
- Explain the qualitative idea of quark lepton family
- Describe the meaning of quantum chromodynamics

5.2 CLASSIFICATION OF ELEMENTARY PARTICLES AND THEIR INTERACTIONS

In the mid-1930s, physicists had a very straightforward understanding of the structure of matter. The proton, the electron, and the neutron were the building blocks of the universe. The photon, the neutrino, and the positron were three other particles that were either known or hypothesised at the time of the discovery of the photon. These six particles were regarded to be the primary elements of matter when taken together. Everyone agreed that this simple picture was insufficient for answering the following important question: because the protons in any nucleus should strongly repel one another due to their charges of the same sign, what is it that holds the nucleus together? No one was able to answer this question with this simple picture. As a result, scientists concluded that this unexplained force had to be much more powerful than anything they had ever observed in nature.

Hideki Yukawa, a Japanese physicist, proposed the first theory explaining the nature of the nuclear force in 1935, an accomplishment that earned him the 1949 Nobel Prize in Physics. To grasp Yukawa's theory, recall the introduction of the field particle, which indicated that each basic force is mediated by an exchange of field particles between interacting particles. Yukawa used this concept to explain the nuclear force, postulating the presence of a new particle whose interaction with nucleons in the nucleus produces the nuclear force. He demonstrated that the force's range is inversely proportional to the mass of this particle and predicted that it would have a mass around 200 times that of the electron. (Yukawa's anticipated particle is not the gluon, which is massless and is now regarded to be the nuclear force's field particle.) Because the new particle would have a mass between the electron and the proton, it was given the name meson (from the Greek *meso*, "middle").

In efforts to substantiate Yukawa's predictions, physicists began experimental searches for the meson by studying cosmic rays entering the Earth's atmosphere. In 1937, Carl Anderson and his collaborators discovered a particle of mass 106 MeV/*c* 2, approximately 207 times the mass of the electron. This particle was thought to be Yukawa's meson. Subsequent experiments, however, showed that the particle interacted very weakly with matter and hence could not be the field particle for the nuclear force. That puzzling situation inspired several theoreticians to propose two mesons having slightly different masses equal to approximately 200 times that of the electron, one having been discovered by Anderson and the other, still undiscovered, predicted by Yukawa. This idea was confirmed in 1947 with the discovery of the **pi meson** (ð), or simply **pion.** The particle discovered by Anderson in 1937, the one initially thought to be Yukawa's meson, is not really a meson. Instead, it takes part in the weak and electromagnetic interactions only and is now called the **muon** (μ).

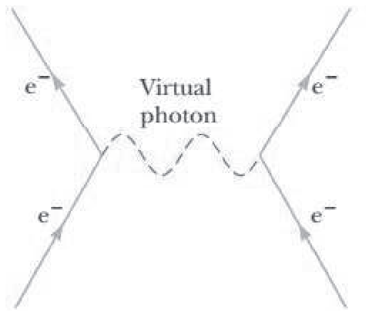

Fig 5.1 *Feynman diagram representing a photon mediating the electromagnetic force between two electrons.*

The pion comes in three varieties, corresponding to three charge states: π^+, π^- and π^0. The π^+ and π^- particles (π^- is the antiparticle of π^+) each have a mass of 139.6 MeV/c^2, and the π^0 mass is 135.0 MeV/c^2. The two muons which exist are μ^- and its antiparticle μ^+.

Pions and muons are very unstable particles. For example, the π^-, which has a mean lifetime of 2.6×10^{-8} s, decays to a muon and an antineutrino. The muon, which has a mean lifetime of 2.2 μs, then decays to an electron, a neutrino, and an antineutrino:

$$\pi^- \rightarrow \mu^- + \bar{\upsilon}$$
$$\mu^- \rightarrow e^- + \upsilon + \bar{\upsilon}$$

For chargeless particles (as well as some charged particles, such as the proton), a bar over the symbol indicates an antiparticle, as for the neutrino in beta decay. Other antiparticles, such as e^+ and μ^+, use a different notation.

The interaction of two particles can be described graphically using a Feynman diagram, which was invented by American physicist Richard P. Feynman. The electromagnetic interaction between two electrons can be seen in Figure 5.1. A Feynman diagram is a qualitative representation of the relationship between time on the vertical axis and space on the horizontal axis. It is qualitative in nature and the exact time and space values are irrelevant; however, the overall appearance of the graph provides a visual depiction of the process.

In the simplest case of the electron–electron interaction as given in Figure 5.1, the electromagnetic force between the electrons is mediated by a photon (the field particle). Notice that the entire interaction is represented in the diagram as occurring at a single point in time. Therefore, the paths of the electrons appear to undergo a discontinuous change in direction at the moment of interaction. The electron paths shown in Figure 5.1 are different from the actual paths, which would be curved due to the continuous exchange of large numbers of field particles.

The photon that transfers energy and momentum from one electron to the next is referred to as a *virtual photon* in the electron–electron interaction because

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it vanishes during the interaction without being observed. We established that the energy of a photon equals $E = hf$, where f is its frequency. As a result, for a system of two electrons initially at rest, the system possesses energy $2m_e c^2$ prior to the release of a virtual photon and energy $2m_e c^2 + hf$ following the release of the virtual photon (plus any kinetic energy of the electron resulting from the emission of the photon). Is this a breach of the law of energy conservation in an isolated system? No; this process does not contradict the law of conservation of energy since the virtual photon has a very short lifetime t, which results in a bigger uncertainty in the system's energy ΔE H" ž$/2\Delta t$ than the photon energy. Thus, under the restrictions imposed by the uncertainty principle, the system's energy is preserved.

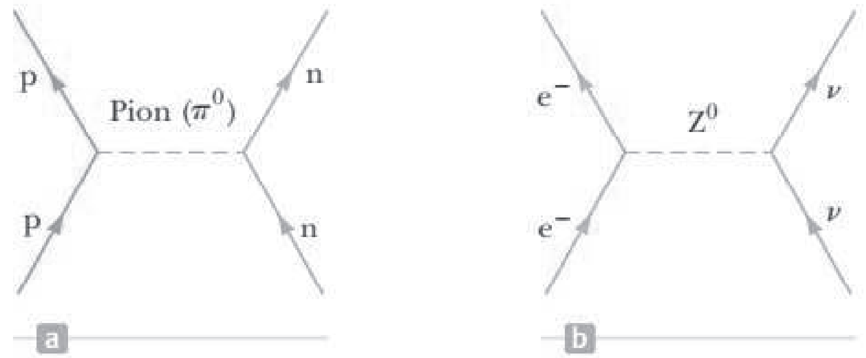

***Fig 5.2** (a) Feynman diagram representing a proton and a neutron interacting via the nuclear force with a neutral pion mediating the force. (This model is* not *the current model for nucleon interaction.) (b) Feynman diagram for an electron and a neutrino interacting via the weak force, with a Z^0 boson mediating the force.*

Now consider a pion exchange between a proton and a neutron according to Yukawa's model (Figure 5.2a). The energy ΔE_R needed to create a pion of mass m_π is given by Einstein's equation $\Delta E_R = m_\pi c^2$. As with the photon in Figure 5.2a, the very existence of the pion would appear to violate the law of conservation of energy if the particle existed for a time interval greater than Δt H"ž$/2\ \Delta E_R$ (from the uncertainty principle), where Δt is the time interval required for the pion to transfer from one nucleon to the other. Therefore,

$$\Delta t \approx \frac{\hbar}{2\Delta E_R} = \frac{\hbar}{2m_\pi c^2}$$

and the rest energy of the pion is

$$m_\pi c^2 = \frac{\hbar}{2\Delta t}$$

Due to the pion's inability to travel faster than the speed of light, the maximum distance d that it can cover in a time interval Δt is $c\Delta t$. As a result of the aforementioned equation and $d = c\Delta t$, we find

$$m_\pi c^2 = \frac{\hbar c}{2d}$$

We know that the nuclear force's range is in the order of 10^{-15} *fm*. Using this value for d in above equation, we estimate the rest energy of the pion to be

$$m_p c^2 \approx \frac{(1.055 \times 10^{-38} J.s)(3.00 \times 10^8\ m/s)}{2(1 \times 10^{15}\ m)}$$

$$= 1.6 \times 10^{11}\,J \approx 100\,MeV$$

which is equivalent to a mass of 100 MeV/c^2 (approximately 200 times the mass of the electron). This number is quite consistent with the mass of the detected pion.

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The preceding thought is rather revolutionary. In effect, it states that a system consisting of two nucleons can transform into two nucleons plus a pion as long as it returns to its original state within a very brief time interval. (Remember that this description is the older historical model, which assumes the pion is the field particle for the nuclear force; the gluon is the actual field particle in current models.) Physicists often say that a nucleon undergoes *fluctuations* as it emits and absorbs field particles. These fluctuations are a consequence of a combination of quantum mechanics (through the uncertainty principle) and special relativity (through Einstein's energy–mass relationship $E_R = mc^2$).

Classification of Elementary Particles

Elementary particles are described using two different forms of statistics, and they are classed according to which statistics they obey.

- Fermi-Dirac statistics are applicable to particles constrained by the Pauli Exclusion Principle; fermions are particles that follow the Fermi-Dirac statistics. Fermions include leptons and quarks. Two fermions cannot exist in the same quantum state. Fermions, in general, are the building blocks of nuclear and atomic structure.
- Bose-Einstein statistics are applicable to all particles that are not excluded by the exclusion principle; these particles are referred to as bosons. There is no restriction on the number of bosons in a given quantum state. In general, bosons operate as a medium for the transmission of forces between fermions; the photon, gluon, W, Z, and Higgs particles are all examples of bosons. Additionally, fundamental particle types have been defined based on additional particle behaviours. Mesons and baryons were previously categorized as strongly interacting particles; it is now known that mesons are composed of quark-antiquark pairs while baryons are composed of quark triplets. Members of the meson class have more mass than that of leptons but less than that of the proton and neutron, but some mesons are heavier than these particles. The proton and neutron are the lightest members of the baryon class, whereas hyperons are the heaviest. There are a handful of particles in the meson and baryon classes that cannot be detected directly because their lives are so brief that they leave no trace in a cloud chamber or bubble chamber. Due to an analogy between their formation and the resonance of an electrical circuit, these particles are referred to as resonances or resonance states.

After researching the structure of atoms, one can conclude that the electron, proton, and neutron are the only three fundamental building blocks of matter. Numerous new nuclear particles have been discovered as a result of studies conducted in part on high-energy cosmic ray particles. They are referred to as elementary particles or subatomic particles. These are elementary particles in the sense that they lack structure. The classification of elementary particles is depicted in Figure 5.3. The elementary particles are classified into two broad categories known as bosons and fermions. Bosons are particles whose intrinsic angular momentum is a multiple of ‘. Fermions are all particles with a spin that is only half integral.

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Baryons

This category includes the proton and particles heavier than protons. Protons and neutrons are referred to as nucleons, while the others are referred to as hyperons. Each baryon has a corresponding antiparticle. If a number called the baryon number is allocated to baryons and a number called the antibaryon number is assigned to antibaryons, then the baryon number does not change during any closed system interaction or decay. This is the conservation law for baryons. Hyperons are a subclass of baryons with a time decay of 10^{-10} seconds and a mass value between the neutron and deuteron. Their decay duration is significantly longer than their formation time (10^{-3}).

Leptons

The electron, photon, neutrino, and muon are all members of this category.

Mesons

These particles have a rest mass of approximately $250m_e$ to $1000m_e$. Mesons are the agents of particle interaction within the nucleus. Pions, kaon, and η- mesons are collectively referred to as **mesons.** Baryons and mesons are jointly called **hadrons** and are the particles of strong interaction.

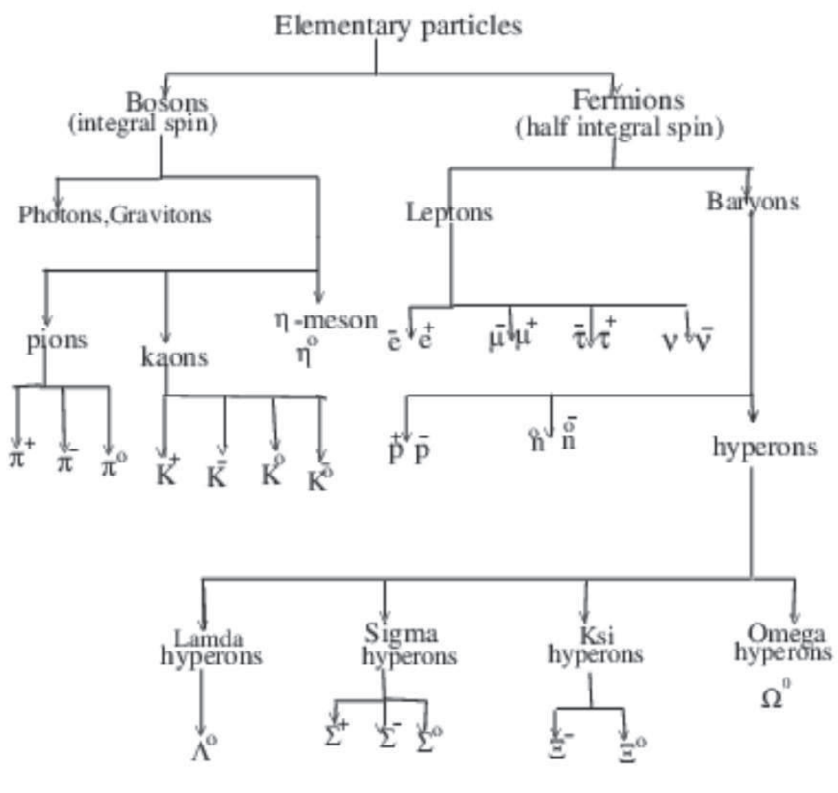

Fig 5.3 *Classification of Elementary Particles.*

5.2.1 Fundamental Interaction

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Fundamental interactions can be defined as the fundamental forces that act between the primary particles that constitute all matter. The fundamental interactions are classified as follows: (i) Strong interaction (ii) Electromagnetic interaction (iii) Weak interaction (iv) Gravitational interaction. All known natural processes (from subatomic to extragalactic scales, i.e., microscopic to macroscopic) can be viewed as manifestations of one or more of these interactions. The following table summarizes the exchange of particles in each of these interactions.

Table 5.1 *Particles exchanged in the interaction.*

Interactions	**Particles Exchanged**
Strong	Mesons
Electro-magnetic	Photons
Weak	Intermediate bosons
Gravitational	Gravitons

For elementary particles and nuclear physics, gravitational forces are irrelevant. Compared to strong and electromagnetic forces, weak forces have a very short range ($<10^{-17}$ m).

1. Strong interaction

The strong interaction is the force that holds nucleons together in the atomic nucleus (nuclear forces). The strong nuclear interaction is not affected by the presence of an electric charge. These interactions have a range of roughly 10^{-15} m. The time period between such interactions is approximately 10^{-23} s.

2. Electromagnetic interaction

It is applicable to all charged particles. Thus, electromagnetism is a charge-dependent phenomenon. The range is limitless, and the interaction is mediated by the photon. Electromagnetic interaction is demonstrated via the production of an electron-position pair from a gamma ray.

3. Weak interaction

All interactions occur within a time range of approximately 10^{-23} s. Their degradation occurs over a period of around 10^{-10} s. Due to the length of time required for particles to respond to such an interaction, the force involved must be very weak in comparison to powerful nuclear forces. This interaction has a range of less than 10^{-17}m. This interaction has a characteristic duration of 10^{-8} s. The weak interaction is responsible for the decay of strange and ordinary particles, as well as for strange particle non-leptonic decays.

Beta decay is an example of weak interaction:

$$n \longrightarrow p + e^- + \bar{\nu}$$

NOTES

4. Gravitational interaction:

This interaction manifests as a strong attraction between all elementary particles over a long distance. Gravity is the first force that all of us discovers. It connects the moon to the earth, keeps the planets in their solar orbits, and connects the stars in our galaxy. Newton defined F as $F = G\, m_1 m_2 / r^2 \sim 2 \times 10^{-34}$ newton. And gravitational attraction has a force of around 2×10^{-49} joule. As a result, we can conclude that it has no effect on particle reactions. It is the least powerful of the four interactions. It possesses a limitless range. Gravitation can be explained in terms of *"gravitons"* interacting. Their mass must be zero, and their velocity must consequently be equal to that of light. The gravitational force is not dependent on the colour, size, charge, velocity, spin, or angular orientation of an object, but on its inertia magnitude. Due to the exceedingly faint gravitational field, gravitons cannot be identified in the laboratory.

Check Your Progress

1. Name the primary elements of matter in the mid-30s.
2. What is a Feynman diagram?
3. What are bosons and fermions?
4. What do you mean by fundamental interaction?

5.3 CONCEPT OF QUANTUM NUMBERS: ISOSPIN, HYPERCHARGE, STRANGENESS, LEPTON AND BARYON NUMBERS

A quantum number is one of numerous integral or half-integral variables that uniquely identify the state of a physical system such as an atom, a nucleus, or a subatomic particle. Quantum numbers broadly refer to discrete (quantized) and conserved properties, such as energy, momentum, charge, baryon number, and lepton number.

It is possible to calculate the energy state and likelihood of finding an electron in an atom by using its primary quantum number, for example. There is a direct correlation between electron energy and distance from the nucleus and increases with increasing main quantum number, which has integral values starting at one. Each electron in an atom can be identified by its principal quantum number and three additional numbers. The atomic nucleus is characterized by a different set of quantum numbers.

Following is an in-depth discussion of quantum numbers and their conservation laws.

Conservation laws of Elementary Particles

Numerous discrete quantum numbers are utilized to classify the numerous fundamental particles. We are already familiar with two of these quantum numbers, those that describe the charge and spin of a particle. Quantum numbers define quantifiable physical qualities and are invariably conserved. We already know that all elementary charges are either zero or one. Bosons are particles with integer spin that obey the Bose-Einstein statistics. Fermions are particles with half odd integer spins that obey the Fermi-Dirac statistics.

1. Baryon number

In the case of a **baryon**, the baryon number B=1 is assigned, and in the case of an anti-baryon, the baryon number B= -1 is assigned. B = 0 is assigned to all other particles. Following a reaction or decay, the law of conservation of baryons dictates that the total baryon number of all particles must remain the same as it was before the reaction or decay. This rule assures that a proton cannot be converted into an electron, although a neutron can be converted into a proton under certain conditions. Baryon conservation protects the proton from decaying into a particle with a lesser mass by preventing it from losing its stability. All normal baryons, such as p^+, n^0, Λ^0, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, Ω, have the baryon of +1. The baryon number -1 is shared by all of the equivalent anti particles, which are referred to as anti-baryons.

All the mesons have baryon number 0.

$\Lambda^0 \longrightarrow p^+ + \pi^-$ *i.e.,* [1 = 1+ 0]

Another illustration of the conservation of baryon number is the following:

$n \longrightarrow p^+ + e^- + \bar{\nu}$ *i.e.,* [1 = 1 + 0 + 0]

Hence the baryon number is conserved.

2. Lepton number

The **Leptons** are said to have a trait known as the *Lepton number* (L). Given the distinction between neutrinos associated with electrons and muons, we add two lepton numbers, L_e and L_μ, which must be preserved separately in particle reactions and decays. L_e = 1 is ascribed to the electron and e-neutrino, while L_e = -1 is assigned to their antiparticles. All other particles have L_e = 0. Also, the number L_μ = 1 is assigned to the **muon** and the **μ – neutrino** and L_μ = -1 to their antiparticles.

$\pi^- + p^+ \longrightarrow \Lambda^0 + K^0$ *i.e.,* [0 + 0 = 0 + 0]

3. Strangeness number

They were formed in high-energy reactions but always in pairs, i.e., when one type of particle is produced, another type of particle is also emitted concurrently. Production of **kaons** and **hyperons**, both of which are classified as odd particles. Decays rapidly due to strong interactions, however this is not observed. Rather than that, they steadily degrade. Due of their peculiar behaviour, they were dubbed odd particles.

$K^0 \longrightarrow \pi^+ + \pi^-$ *i.e.,* [+ 1 = 0 + 0]

Here the LHS is 1 and RHS is 0. So, the reaction is not conserved for strangeness.

In all processes mediated by strong and electromagnetic interactions, S is observed to be conserved. The conservation principle results in the multiple production of particles with S not equal to 0. An example is the proton-proton collision.

$$P^+ + P^+ \longrightarrow \Lambda^0 + K^0 + P^+ + \pi^+$$

strangeness number S; $0 + 0 \longrightarrow (-1) + 1 + 0 + 0.$

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A weak interaction, on the other hand, can cause S to alter in an event that is ruled by it. It is through the weak contact that the decays of kaons and hyperons take place, and as a result, they are incredibly sluggish. The weak interaction, on the other hand, is incapable of allowing S to change by more than ±1 during a decay.

$$\Sigma^+ \rightarrow \Lambda^0 + e^+ + \nu_e \qquad [\,-1 = -1 + 0+0\,] \;;\quad \Delta S=0$$

$$\Sigma^- \rightarrow n + e^- + \bar{\nu}_e \qquad [\,-1= 0+0+0\,] \;;\quad \Delta S= -1$$

$$\pi^- + p \rightarrow \Lambda^0 + K^0 \qquad [\,0+ 0 = -1 +1\,] \;;\quad \Delta S=0$$

4. Isospin and Isospin quantum number

In terms of strong interactions, the neutron and proton are two equal-mass states of a nucleon doublet. Multiplets of particles are discovered. For instance, **η^0 (eta-meson)**, **Ω^- (omega hyperons)**, **Λ^0 (lamda hyperon)**. Doublet: *p, n*, triplet, **π^+, π^-, π^0 (pions or Pi-mesons)**. It is easy to think of a multiplet's members as representing the various charge states of a single fundamental entity. It has been advantageous to classify each multiplet according to the number of charge states it shows by a number *I* such that the state's multiplicity is given by $2I+1$. Thus, $I = ½$ is ascribed to the nucleon multiplet, and its $2 \times ½ + 1 = 2$ states are the neutron and proton. $I = 1$ for the pion multiplet, and its $2 \times 1 + 1 = 3$ states are π^+, π^-, π^0.

Isospin can be represented mathematically as a vector *I* in "isospin space" whose component in any specified direction is defined by a quantum number commonly designated I_3. I_3 has the following potential values: $I, I-1, I-2,\ldots\ldots,$ $0, \ldots.. -(I-1), -I$. Thus, if *I* is half integral, I_3 is half integral, and if *I* is integral, I_3 is integral or zero.

$I = ½$ for the nucleon, which indicates that I_3 can be either +1/2 or -1/2; the former is assumed to represent the proton, while the latter is assumed to represent the neutron.

Furthermore, for the pion triplet $I = 1$, $I_3 = +1$ for +, $I_3 = 0$ for π^0 and –1 for π^-. The charge of a meson or baryon is proportional to its baryon number ***B***, strangeness number ***S***, and isospin component ***I_3*** by the formula.

$$q = e\left(1_3 + \frac{B+S}{2}\right) = e\left(I_3 + \frac{Y}{2}\right)$$

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Class	Name	Symbol	Spin	B	Le	Lμ	S	Y	I
LEPTON	e- neutrino	ν_e	½	0	+ 1	0			
	μ – neutrino	ν_μ	½	0	0	+1			
	Electron	e^-	½	0	+ 1	0			
	Muon	μ^-	½	0	0	+1			
MESON	Pion	$\bar{\pi}, \pi^+$ π^0	0	0	0	0	0	0	1
	Kaon	K^+ K^0	0	0	0	0	+ 1	+1	1/2
	η meson	η^0	0	0	0	0	0	0	0
BARYON	Nucleon	*p,n*	½	+ 1	0	0	0	+1	1/2
	Λ hyperons	Λ^0	½	+ 1	0	0	-1	0	0
	Σ hyperons	Σ^+ Σ^0 Σ^-	½	+ 1	0	0	-1	0	1
	Ξ hyperons	Ξ^0 Ξ^-	½	+ 1	0	0	-2	-1	1/2
	Ω hyperons	Ω^-	3/2	+ 1	0	0	-3	-2	0

5. Hypercharge

In intense contact, a quantity called **hypercharge (Y)** is conserved. The hypercharge of a particle is equal to the product of its strangeness and baryon numbers, $Y = S + B$. Because $B=0$ for mesons, hypercharge equals weirdness.

5.4 INVARIANCE

Many of nature's most profound thoughts reveal themselves in the form of symmetry. During a physical experiment, symmetry suggests that something is conserved, or that something remains constant, throughout the experiment. As a result, conservation rules and symmetries have a strong relationship.

Conservation Laws and Symmetry

A significant set of conservation laws is connected to parity (P), charge conjugation (C), and time reversal symmetries (T).

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Charge Conjugation Symmetry

Charge conjugation is a symmetry operation that replaces each particle in a system with its antiparticle. Charge parity (C) is conserved if the antisystem, or antimatter counterpart, exhibits the same physical phenomena. For instance, if the proton in a hydrogen atom is replaced by an antiproton and the electron by a positron, the resulting antimatter atom behaves identically to an ordinary atom. Indeed, the weak contact does not conserve C.

5.4.1 Concept of Variation Laws and Selection Rules in Relation to Particle Production

Let us discuss the particles and anti-particles.

1. Electron and Positron

Antiparticles are defined as particles that are opposed to one another. They have the same mass and spin as one other, but they have the opposite charge. When they come into contact with one other, they annihilate each other by releasing photons into the environment. Dirac was the first to anticipate the existence of an antiparticle for the electron, and he was correct. Anderson made the discovery of the positron in 1932.

2. Proton and Antiproton

It was Segre, Chamberlain, and their coworkers that first developed the antiproton in 1955. Antiprotons were created by hitting protons in a target with protons with energies of 6 GeV or higher, thereby encouraging the reaction to occur.

$$p + p + (energy) \longrightarrow p + p + p + \bar{p}$$

Antiprotons have a strong interaction with matter and annihilate when they come into contact with protons.

$$p + \bar{p} \longrightarrow \pi^+ + \pi^- + \pi^+ + \pi^- + \pi^o$$

3. Neutron and Antineutron

Cork, Lamberton, and Wenzel discovered the *antineutron*, the neutron's antiparticle, in 1956. The antineutron's nature is unknown. Both the neutron and the antineutron are neutral and have the same mass. However, because the neutron is meant to have a specific internal charge distribution, it is believed that the antineutron will have the opposite internal charge distribution. Antineutrons are rapidly annihilated, either by a proton or a neutron, producing many pions in the process. If an antineutron is not completely destroyed by a nucleon, it decays via the reaction.

$$\boldsymbol{p} + \bar{p} \longrightarrow \bar{n} + \boldsymbol{n}$$

4. Neutrino and Antineutrino

The neutrino has a finite energy and momentum in flight. It travels with the speed of light *c*. It does not cause ionization on passing through matter. The antiparticle of neutrino is antineutrino.

The contrast between neutrino (υ) and antineutrino () is especially fascinating. The neutrino's spin is in the opposite direction of its speed. The neutrino spins in the anti-clockwise direction. However, the antineutrino's spin is in the same direction as its speed; it spins clockwise. Thus, the neutrino travels across space like a left-handed screw, but the antineutrino travels like a right-handed screw. Thus, the neutrino has a "left-handed" helicity, whereas the antineutrino has a "right-handed" helicity, i.e., the neutrino and antineutrino are identical except for their helicity.

NOTES

It is conventional to refer to the particle associated with a positron as a neutrino, while the particle associated with an electron is referred to as an antineutrino. A neutrino has virtually minimal interaction with matter due to its lack of charge and magnetic moment. This interaction is negligible.

$$n \longrightarrow p + e^{-} +$$

$$p \longrightarrow n + e^{+} + \upsilon$$

Antimatter

It has long been useful in atomic physics to think of an atom as being constituted of additional nuclear electrons and a nucleus. A positron and an antiproton can combine to generate an anti-hydrogen atom. The spectrum of anti-hydrogen would be comparable to that of regular hydrogen. Indeed, everything was constructed entirely of antiparticles from a collection of anti-protons, anti-neutrons, and positrons. Particle-antiparticle annihilation would result in a significant amount of energy being released.

Check Your Progress

5. What is a quantum number?
6. Why do the neutrino have a left-handed helicity and antineutrino a right-handed helicity?

5.5 DECAY OF CHARGE CONJUGATION

In this section, we will discuss the decay of charge conjugation.

CP Violation: The weak force, which is responsible for events like the radioactive disintegration of atomic nuclei, violates the combined conservation laws of charge conjugation (C) and parity (P) in particle physics. Charge conjugation is a mathematical process that turns a particle into an antiparticle by reversing the electric charge sign, for example every charged particle has an oppositely charged antimatter counterpart, or antiparticle, according to charge conjugation. An electrically neutral particle's antiparticle can be identical to the particle, as in the instance of the neutral pi-meson, or it can be unique, as in the case of the antineutron. Parity, or space inversion, is the reflection of a particle's or particle system's space coordinates via the origin; i.e., the three space dimensions x, y, and z become -x, -y, and -z, respectively. In more concrete terms, parity conservation means that up and down, left and right, are indistinguishable in the sense that an atomic nucleus emits decay products up as well as down, and left as well as right.

According to this assumption, all interactions between particles were thought to be perfectly symmetrical in terms of charge conjugation or parity—that is, these two qualities were always conserved in particle interactions. This has now been proven to be incorrect. Reversing time (T) was shown to have the same effect as doing so, as was reversing motion (V). When a motion is permitted by the laws of

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physics, the reversed motion is likewise permitted. Invariance under time Discoveries made in the mid-1950s have changed the way scientists think about invariance of C, P, and T. Two or three pi-mesons decaying from two or three charged K-mesons motivated theoretical physicists Chen Ning Yang and Tsung-Dao Lee to investigate the experimental foundation for the conservation of parity. For the first time, in 1956, they found no evidence of parity invariance in weak interactions. Particle decays such as nuclear beta decay, which take place via the weak force, do not preserve the conservation of parity. Charge conjugation symmetry was also violated during these decay processes, as shown by these experiments.

A quantitative theory establishing coupled CP as a symmetry of nature was developed after it was discovered that the weak force does not retain charge conjugation or parity independently. Because CP is invariant, it was assumed that time reversal T would be as well. The electrically neutral K-meson, which ordinarily decays via the weak force to generate three pi-mesons, decayed a fraction of a time into only two of these particles in 1964, according to a team led by American physicists James W. Cronin and Val Logsdon Fitch. Assuming that the long-held CPT theorem is correct, then CP violation implies non-conservation of T. When applied in any order, charge conjugation, parity, and time reversal should ensure that all interactions remain stable. There exists a precise balance in all fundamental interactions, and this is known as the CPT symmetry.

Quarks, the fundamental building blocks of K-mesons, exert a weak force on one another that causes this phenomenon. The weak force appears to act on a quantum mixture of two sorts of quarks, rather than on a pure quark state, as identified by the "flavour" or type of quark. Theoretical physicists Makoto Kobayashi and Toshihide Maskawa from Japan hypothesised in 1972 that the Standard Model of particle physics would include an inherent prediction of CP violation if there were six types of quarks. For their "finding of the origin of the broken symmetry which predicts the existence of at least three family groups in nature," Kobayashi and Maskawa were awarded the Nobel Prize in Physics in 2008. Quark quantum mixing allowed them to discover uncommon decays that would break CP symmetry. The bottom and top quarks, members of the third generation of quarks, were discovered in 1977 and 1995, respectively, confirming their predictions.

The Kobayashi-Maskawa theory appears to be supported by experiments with neutral K-mesons, however the effects are negligible. For B-mesons, CP violation is projected to be more significant than for K-mesons, because the B-mesons have a lower-energy quark than the K-mesons. B-mesons, heavier than K-mesons, are being tested in experiments at facilities that can create a significant quantity of them. B-mesons decaying into muons rather than anti-muons was first noticed in 2010 by researchers at the Fermi National Accelerator Laboratory in Batavia, Illinois.

There are theoretical implications to CP violation. Physicists can distinguish between matter and antimatter with certainty because of the violation of CP symmetry. The study of the cosmos may be profoundly affected by the distinction between matter and antimatter. In physics, one of the unanswered puzzles is why the cosmos is composed primarily of matter. The observed imbalance or asymmetry in the matter-antimatter ratio may have been caused by the occurrence of CP violation in the initial seconds following the big bang—the catastrophic explosion that is supposed to have resulted in the genesis of the universe.

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Check Your Progress

7. What is charge conjugation?
8. How can the physicists distinguish between matter and antimatter with certainty?

5.6 PARITY INVARIANCE AND TIME REVERSAL WITH SIMPLE APPLICATION IN PARTICLE PHYSICS

In this section, we will study about the parity invariance and time reversal along with application in particle physics.

Conservation of parity

Parity is a term that refers to the symmetry of the wave function used to describe the system. When the coordinates (x, y, z) are replaced by (-x, -y, -z), the system has a parity of + 1. When the sign of the wave function is flipped, the system has a parity of -1. If we write ψ (x, y, z) = P ψ (-x, -y, -z), we can consider *P* to be a quantum number describing ψ with values between +1 and -1. The overall parity number does not change during a reaction in which parity is conserved.

By converting the coordinates (x, y, z) to (-x, -y, -z), a right-handed coordinate system can be converted to a left-handed coordinate system. In terms of symmetry, conservation of parity means that in every circumstance where parity exists, the description of the reaction will remain unchanged if the term "left" is substituted for "right" and vice versa. This means that such replies cannot provide a signal as to which direction is right or left. Until 1956, it was assumed that all natural processes followed the law of conservation of parity. Yang and Lee showed that parity was not conserved in reactions involving the weak contact, and that studies might be designed to definitively discriminate between right and left. Indeed, it is discovered that parity conservation holds true only in strong and electromagnetic interactions.

Time Reversal Symmetry

Parity in time *T* denotes how a wave function behaves when *t* is replaced by *–t*. Time reversal is the symmetry operation that corresponds to the conservation of time parity. Time reversal symmetry indicates that the direction of time is irrelevant, and hence that the inverse of any process that is capable of occurring is also capable of occurring. In other words, if time reversal symmetry holds, it is difficult to determine whether a motion picture of an event is being played forward or backward when viewed. Prior to 1964, it was assumed that time parity *T* was conserved in all interactions. In 1964, it was revealed that one of the K^0's forms, kaon, can decay into π^+ and π^-;

$K^0 \longrightarrow \pi^+ + \pi^-$ *i.e.,* **[+ 1 = 0 + 0]**

It is in violation of parity in time's (T) conservation. Thus, the symmetry of events observed during time reversal does not appear to be universal.

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CPT Theorem

The combined symmetry operation, which reverses the antimatter mirror image of a system, enables the *CPT* invariance to be tested. The data is overwhelming in favour of *CPT* conservation. Because *CPT* is conserved, each process has an antimatter mirror-image counterpart that occurs in reverse. This particular symmetry appears to hold true for all interactions, despite the fact that its component symmetries occasionally fail on their own.

Check Your Progress

9. Define the term 'parity'.
10. What is isospin?

5.7 ELEMENTARY IDEA OF SU(2) AND SU(3) AND RESULTS OF GROUP THEORY

Let us discuss the elementary idea of SU(2) and SU(3) along with the results of group theory.

Unitary Symmetry (SU(2) Symmetry)

As far as the nuclear force is concerned, we know that the proton and neutron are identical, yet their electromagnetic interactions are distinct. Thus, in the absence of an electromagnetic field, one can envision a group of symmetry operators capable of transforming a neutron into a proton (or a proton into a neutron). The proton and neutron would therefore serve as the group's fundamental representations. The presence of such symmetry implies that something is constant in the presence of a strong interaction. This is referred to as isospin, and it is ½ for both the proton and neutron. The isospin component, T_3, is + ½for the proton and - ½ for the neutron. Thus, the operators of the symmetry group alter the coordinates of isospin in such a way that the sign of T_3 is reversed. Additionally, it can be written as: the strong interactions are expected to be invariant through isotopic spin space rotations. Isospin conservation requires a special type of unitary symmetry known as U(2), which can be written as a set of 2×2 matrices. This group can be reduced to a special unitary group denoted by the symbol SU(2), abbreviated as SU2. It is unique in that a limitation reduces the number of operators in the group by unity. In this situation, the two dimensions correspond to the two fundamental states that comprise the fundamental representation. The constraint imposed by the restriction of limiting the number of operators from $2 \times 2 = 4$ to three. As a result, the group is considered to have three generators.

By employing the SU(2) group algebra, it is possible to demonstrate that all irreducible representations of the symmetry group contain a multiplet of $2T + 1$ states. The multiplet's members all have the same isospin *T* and are otherwise identical except for charge. If the symmetry were perfect, that is, if isospin was perfectly conserved, the components of a multiplet would differ in charge and T_3. The electromagnetic interaction violates the SU(2) symmetry because conservation of isospin does not apply.

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Eightfold Way [SU(3) Symmetry]

Due to the fact that the SU(2) group cannot fit the hypercharge quantum number, the more general SU(3) theory was utilized, which incorporates SU(2). SU(3) denotes a three-dimensional special unitary group. Three dimensions refers to the three fundamental states that comprise the fundamental representation in this instance. There are in general $3 \times 3 = 9$ operators in a three-dimensional unitary group, but the restriction of "*special*" decreases the number to eight. After then, the group is claimed to have eight generators. Three of the generators are related with isospin's three components, as in SU(2), and a fourth with hypercharge. The remaining four utilize hypercharge in a somewhat different manner.

The group algebra revealed that the SU(3) symmetry should result in the formation of six super multiplets with 1, 8, 8, 10, and 27 members. The multiplet is equivalent to the 10, but with opposite-sign hypercharges. The parity and intrinsic spin of members are same in each of these super multiplets, but the hypercharge and isotopic spin are not identical. Among the aforementioned groups, those with 8 and 10 members are particularly noteworthy.

In the case for B=0 we may form particles anti particle states in a 3×3 array. The matrix may include up to 8 states and is referred to as an octet, due to the fact that mesons are produced from fermions particle-antiparticle pairs and hence have an odd parity. These 8 particles with $B=0$ and $J^{\pi} = 0^{-}$ should be arranged as follows and the schematic representation is given in the Figure 5.4.

One triplet with	Y=0, T=1		π^-, π^0, π^+
One double with	Y=1, T= ½		K^0, K^+
One double with	Y= -1, T= ½	8 members	$\overline{K^0}, K^-$
One single with	Y= 0, T=0		η^0

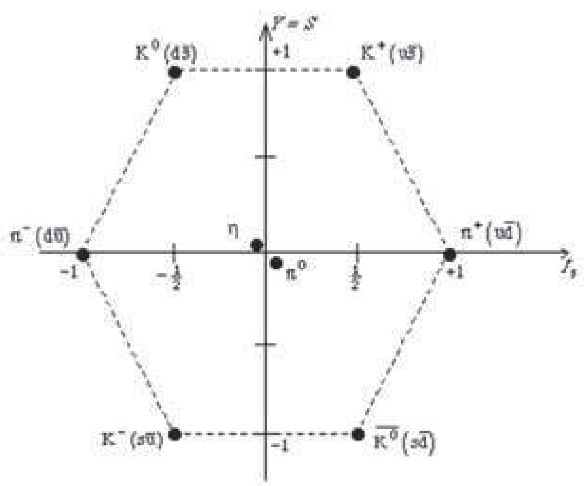

Fig 5.4 *Meson Octet in Eightfold-Way representation.*

Paradoxes of Quark Model

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- Quarks have fractional charges while all observed particles have integer charges. At least one of the quarks is stable. None has been found.
- Hadrons are exclusively made out *q*; *q q q* bound states. In other word, *q q; q q q q* states are absent.
- The quark content of the baryon X^{*++} is u u u. If we choose the spin states of these three, this will lead to violation of Pauli Exclusion Principle.

5.8 GELL-MANN-OKUBO MASS FORMULA AND APPLICATION TO MASS SPECTRA OF PARTICLES

Since SU(3) is not an exact symmetry, we want to see whether we can understand the pattern of the SU(3) breaking. Experimentally, SU(2) seems to be a good symmetry, we will assume isospin symmetry to set $m_u = m_d$: Assume that we can write the hadron masses as linear combinations of quark masses.

• For spin-0, odd parity mesons, (0" meson)

Assume that the meson masses are linear functions of quark masses:

$$m_\pi^2 = \lambda(m_o + 2m_u)$$
$$m_\kappa^2 = \lambda(m_o + m_u + m_s)$$
$$m_\eta^2 = \lambda(m_o + \frac{2}{3}(m_u + 2m_s)$$

where and m_0 and λ are some constants with mass dimension. Eliminate the quark masses we get

$$4m_\kappa^2 = m_u^2 + 3m_\eta^2$$

Experimentally, we have 4 H'' 0.98 *(GeV)*^{2} while + 3 H'' 0.92 *(GeV)*^{2} This seems to show that this formula works quite well.

• For spin $\frac{1}{2}$, even parity baryons, ($\frac{1}{2}$ *baryon*)

Here we assume that the meson masses are linear functions of quark masses;

$$m_N = (m_o + 3m_u)$$
$$m_\Sigma = m_\Lambda = (m_o + 2m_u + m_s)$$
$$m_\Xi = (m_o + 2m_u + 2m_s)$$

where m_0 is a constant with mass dimension. By eliminating the quark masses we get the Gell-Mann Okubo mass formula for baryons with spin-$\frac{1}{2}$ as

$$\frac{m_\Sigma + 3m_\Lambda}{2} = m_N + m_\Xi$$

Expermentally, $\frac{m_\Sigma + 3m_\Lambda}{2} \approx 2.23\, GeV$ and $m_N + m_\Xi \approx 2.25\, GeV$

• For spin- $\frac{1}{2}$, even parity baryons, ($\frac{3^*}{2}$ *baryon*)

The mass relation here is quite simple. This sometimes is referred to as **equal spacing rule**. In fact when this relation is derived the particle &! has not yet been found and this relation is used to predicted the mass of and subsequent finding provide compelling evidence for SU(3) symmetry. for baryons with spin-3/2, the Gell-Mann Okubo mass formula as

$$m_\Omega - m_\Xi = m_\Xi - m_\Sigma = m_\Sigma - m_N$$

5.9 QUALITATIVE IDEA OF QUARK LEPTON FAMILY

In this section, we will discuss the qualitative idea of quark lepton family.

The Quark Model

The quark model was proposed in 1964 by Murray Gell-Mann and G. Zweig. This theory is based on the notion that hadrons are constructed from a finite number of fundamental components known as quarks. The first three quarks were designated ***u*** (for "*up*"), ***d*** (for "*down*"), and ***s*** (for "*strange*").

Quark	symbol	charge	spin	B	S	I	I_3
Up	u	+2/3	½	1/3	0	½	+1/2
down	d	-1/3	½	1/3	0	½	-1/2
strange	s	-1/3	½	½	-1	0	0

Fig 5.5 *Quarks*

u quark has electric charge + e and strangeness 0.

d quark has electric charge – e and strangeness 0.

s quark has electric charge – e and strangeness – 1.u quark has electric charge + $\frac{2}{3}$ e and strangeness 0.

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d quark has electric charge – $\frac{1}{3}$ e and strangeness 0.

s quark has electric charge – e and strangeness – 1.

Each quark has a baryon number of B = $\frac{1}{3}$.

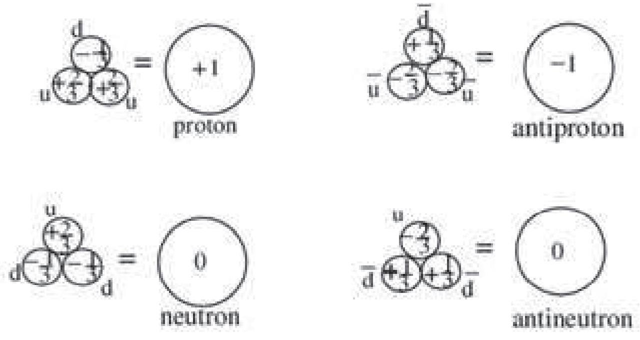

Fig 5.6 *Constituents of proton and neutron in terms of quark*

Each quark has an antiquark associated with it (). The magnitude of each of the quantum numbers for antiquarks is the same as that of quarks, but the sign is different.

Compositions of hadrons according to the quark model

Hadrons may be baryons or mesons. A baryon is made up of three quarks.

The proton, for example, is made up of two u quarks and one d quark (*uud*). The electric charges of these quarks are + 2/3, + 2/3, and – 1/3, giving a total value of + 1. For a sum of + 1, the baryon numbers are + 1/3, + 1/3, and + 1/3. The numbers for strangeness are 0, 0 and 0 for a total strangeness of 0. The quantum numbers for the proton are all in agreement. Quark models of the proton, antiproton, neutron and antineutron. Electric charges are measured in *e* units.

One quark and one antiquark make a meson. The π^+ meson, for example, is made up of one quark, either *u* or *d*, and one antiquark, either . These quarks have electric charges of + 2/3 and + 1/3, for a total of +1 and a baryon number of 0. The numbers for weirdness are 0 and 0 for a total of 0. All of these are in agreement with the pi-quantum meson's numbers.

The half-integral spins of baryons and the 0 or 1 spins of mesons are entirely due to quarks, which have spins of ½.

The various quarks and their antiquarks can be used to explain all known hadrons. Five hadrons' quark contents and how they account for their observed charges, spins, and strangeness values are given in Table 5.2.

Table 5.2 *Five hadrons' quark contents and how they account for their observed charges, spins, and strangeness values*

Hadron	*Quark content*	*Baryon number*	*Charge, e*	*Spin*	*Strangeness*
π^+	$u\bar{d}$	$\frac{1}{3}-\frac{1}{3}=0$	$+\frac{2}{3}+\frac{1}{3}=+1$	$\uparrow\downarrow=0$	$0+0=0$
$K+$	$u\bar{s}$	$\frac{1}{3}-\frac{1}{3}=0$	$+\frac{2}{3}+\frac{1}{3}=+1$	$\uparrow\downarrow=0$	$0+1=+1$
$p+$	uud	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=+1$	$+\frac{2}{3}+\frac{2}{3}+\frac{1}{3}=1$	$\uparrow\uparrow\downarrow=\frac{1}{2}$	$0+0+0=0$
n^0	ddu	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=+1$	$-\frac{1}{3}-\frac{1}{3}+\frac{2}{3}=0$	$\downarrow\downarrow\uparrow=\frac{1}{2}$	$0+0+0=0$
$\Omega-$	sss	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=+1$	$-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}=-1$	$\uparrow\uparrow\uparrow=\frac{3}{2}$	$-1-1-1=-3$

Coloured Quarks and Gluons

The quark model had flaws, one of which was the Ω^- hyperon. Three identical *s* quarks were supposed to be contained within (*sss*). This is a violation of the Pauli Exclusion Principle, which stipulates that no two or more fermions can occupy the same quantum state. Protons, neutrons, and other elementary particles with two identical quarks would likewise break this concept. This challenge can be overcome by endowing quarks with a new attribute. This new feature can be thought of as an extra quantum number that can be used to designate the three otherwise identical quarks in the Ω^-. If this additional quantum number has one of three potential values, we can restore Pauli's principle by assigning a different value to each quark for this new quantum number, dubbed colour. The three colours are denoted by the letters red (R), blue (B), and green (G). For instance, the Ω^- would then be S_R, S_B, and S_G. Anti-red (*R*), anti-blue (*B*), and anti-green (*G*) are the antiquark colours (*G*).

A necessary component of the quark model with colour is that all observable meson and baryon states are "*colourless*", that is, either colour or anti-color combinations of mesons, or equal mixtures of R, B, and G for baryons.

Given that hadrons appear to be made of quarks, the strong interaction between hadrons should eventually be traced to a quark-quark interaction. The force between quarks can be described as an exchange force, mediated by the interchange of massless spin –1 particles called gluons. There have been eight gluons proposed. The field that holds the quarks together is a field of colour. Colours are to quarks' strong interaction what electric charge is to electrons' electromagnetic interaction. It is the inherent strong "*charge*" that gluons carry. Thus, gluons must be represented as mixtures of a primary colour and a possible secondary anti-color. The gluons are massless and carry their colour and anti-colour qualities in the same way that other particles do. For instance, the exchange of a gluon RB by red and blue quarks. In effect, the red quark converts its redness to a gluon and gains blueness through the simultaneous emission of anti-blueness.

On the other hand, the blue quark absorbs the R gluons, wiping out its blueness and obtaining a red colour.

Charm, Bottom, and Top

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The charmed quark was proposed as a possible explanation for the non-observed inhibition of certain decay processes. The processes would have progressed at detectable rates with only three quarks and should have been noticed. The charm quark has a charge of *e*, a strangeness of 0, and a quantum number of charm of + 1. The charm of other quarks is zero.

Table 5.3 *Charm, Bottom and Top*

Generation	Quark	Symbol	Charge, e	Strangeness	Charm
1	Up	u	$+\frac{2}{3}$	0	0
	Down	d	$-\frac{1}{3}$	0	0
2	Charm	c	$+\frac{2}{3}$	0	+1
	Strange	s	$-\frac{1}{3}$	-1	0
3	Top	t	$+\frac{2}{3}$	0	0
	Bottom	b	$-\frac{1}{3}$	0	0

Quantum Numbers: The quarks have quantum numbers. The *s*-quark has a quantum number called strangeness. In the strong and electromagnetic interactions, the *C*, *B*, and *T* quantum numbers are preserved, but in the weak interactions, they change by one unit. This indicates that in strong and electromagnetic interactions, the amount of quarks minus antiquarks for each *s*, *c*, *b*, and *t* must be constant, whereas in weak interactions, quark flavor changes with the preferred sequence $t \to b \to c \to s$. Because three quarks are required to create a baryon, the baryon number for all quarks is 1/3. The quarks quantum numbers are summarized in table.

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Table 5.4 *Quark Quantum Numbers and Properties.*

Quantum Number	*u*	*d*	*s*	*c*	*b*	*t*
Charge	2/3	-1/3	-1/3	2/3	-1/3	2/3
Mass (GeV/c^2)	0.39	0.39	0.51	1.55	~ 5	~ 30
Spin in ***h***	1/2	1/2	1/2	1/2	1/2	1/2
Isospin ***I***	1/2	1/2	0	0	0	0
Isospin Component I_3	1/2	-1/2	0	0	0	0
Baryon number ***B***	1/3	1/3	1/3	1/3	1/3	1/3
Strangeness ***S***	0	0	-1	0	0	0
Charm $\mathcal{C}$	0	0	0	1	0	0
Botton $\mathcal{B}$	0	0	0	0	-1	0
Top $\mathcal{T}$	0	0	0	0	0	1

T_3=1/2 and -1/2 for the up and down quarks, respectively, because the isospin quantum number *T* is ½. The strange quark's and beauty quark's quantum number *S* are both -1. It's a 1 because of the charm and the top quarks. The isospin is a quantum number connected to the u-d quark difference, whereas the hypercharge is a quantum number related to quark strangeness and baryon number. The degeneracy is broken by the colour quantum number, which permits up to three quarks of the same flavour to occupy a single quantum state.

Quark Masses: The *u* and *d* quarks, which have the identical mass of 0.39 GeV/c^2, are the least massive of the six quarks. As a result, the lightest baryons, nucleons, Š particles, and lightest mesons, pions, must be formed entirely of these two quarks. The *s* quark has a higher mass of 0.51 GeV/c^2. It carries a strangeness quantum number and is a required constituent of strange particles (with non-zero strangeness), such as *K*-mesons and baryon *A*. The *c*-quark, with a rest mass of 1.65 GeV/c^2, is even more enormous. The rest mass of the *b*-quark is roughly 5 GeV/c^2.

Mesons and Baryons

Hadrons are constructed entirely of six quarks and their antiquarks. The quarks' properties are inferred from the mesons and baryons' properties. To determine the masses of quarks from known hadron masses, we must first determine the strength of the interaction between quarks within the hadron. Hadrons are classified

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into two distinct classes: baryons and mesons. Because baryons are fermions, quarks are also fermions. Due to the fact that the quark cannot exist as a free particle, the hadron family's lightest fermion must be composed of three quarks. Thus

$$|p\rangle = |uud\rangle \quad \text{and} \quad |n\rangle = |u\,dd\rangle.$$

$$Q_p = \frac{2}{3} + \frac{2}{3} + \left(\frac{-1}{3}\right) = 1; \qquad Qn = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0.$$

Elementary Particle Symmetries

It has been demonstrated that the conservation law, in general, reflects an invariance that corresponds to a suitable symmetry operation. The group from which the theory gets its name is composed of the set of operators that represent the symmetry. The irreducible representations of a group are a collection of states, quantities, or objects that are subject to symmetry operations. Thus, using the appropriate group operation, any of these states can be transformed into another in the same representation. The fundamental representation is the one that contains the fewest possible states for a given group.

If the system is invariant with regard to movement in space, linear momentum is conserved. If invariance exists with respect to angular displacement, angular momentum is conserved; if invariance exists with respect to time, energy is conserved.

The simplest unitary group, *U (1)*, contains transformations that modify only particle wave functions by adding a phase factor. Charge *Q*, baryon number *B*, lepton number *L*, and hypercharge *Y* are conserved as a result of invariance under such changes.

Multicolored Quarks: Soon after the quark notion was proposed, scientists realised that certain particles possessed quark compositions that violated the exclusion principle. We used the principle of exclusion to electrons in atoms. The principle is more general, however, and applies to all particles with half-integral spin ($\frac{1}{2}, \frac{3}{2}$ etc.), which are collectively called fermions. Because all quarks are fermions having spin , they are expected to follow the exclusion principle. The Ω^- (*sss*) baryon, for example, appears to break the exclusion principle because it includes three odd quarks with parallel spins, giving it a total spin of $\frac{3}{2}$. In violation of the exclusion principle, all three quarks have the same spin quantum number. Other examples of baryons made up of identical quarks having parallel spins are the Δ^{++} (uuu) and the Δ^- (ddd).

It was proposed that quarks have an additional attribute called colour charge to alleviate this difficulty. This attribute is comparable to electric charge in many ways, although it comes in six types rather than two. Quarks have the colours red, green, and blue ascribed to them, while antiquarks have the colours anti-red, anti-green, and antiblue. As a result, the colours red, green, and blue are used as "quantum numbers" for the quark's colour. To meet the exclusion principle, each baryon's three quarks must be of different colours. Look again at the quarks in the baryons in Figure 6 and notice the colors. The three colours "*neutralize*" to white.

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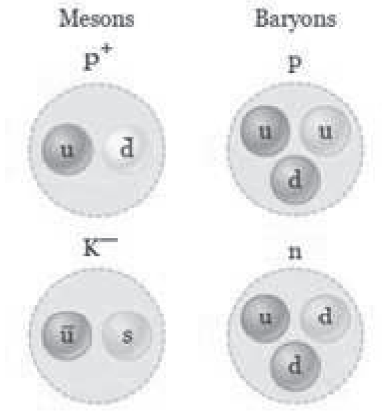

Fig 5.7 *Quark composition of two mesons and two baryons.*

A quark and an antiquark in a meson must be of a color and the corresponding anti-color and will consequently neutralize to white, similar to the way electric charges 1 and 2 neutralize to zero net charge. (See the mesons in Figure 5.7) The apparent violation of the exclusion principle in the V^2 baryon is removed because the three quarks in the particle have different colors.

Because each of the six quarks comes in three colours, the new property of colour boosts the number of quarks by a factor of three. Although the quark model's concept of colour was created to satisfy the exclusion principle, it also provided a better explanation for certain experimental data. The updated theory, for example, accurately predicts the lifespan of the p^0 meson.

5.9.1 Quantum Chromodynamics

Quantum chromodynamics, or QCD, is the name given to the theory of how quarks interact with one another, similar to *quantum electrodynamics* (the theory of the electrical interaction between light and matter). In quantum chromodynamics, each quark is considered to have a colour charge, which is analogous to electric charge. The **colour force** is the strong interaction that exists between quarks. As a result, the terms "*strong force*" and "*colour force*" are frequently interchanged.

The nuclear contact between hadrons is mediated by massless field particles known as **gluons**, as previously explained. The nuclear force is a side effect of the strong force between quarks, as previously stated. The gluons are the strong force's mediators. The colour of a quark can vary when it emits or absorbs a gluon. A blue quark emitting a gluon, for example, may become a red quark, while a red quark absorbing this gluon may become a blue quark. The electric force between charges and the colour force between quarks are analogous particles with the same color repel, and those with opposite colors attract. Therefore, two green quarks repel each other, but a green quark is attracted to an anti-green quark. The attraction between quarks of opposite color to form a meson (q) is indicated in Figure 5.8a. Differently coloured quarks also attract one another, although with less intensity than the oppositely colored quark and antiquark. As seen in Figure 5.8b, a cluster of red, blue, and green quarks all attract one another to create a baryon. As a result, each baryon is made up of three quarks of three different colours.

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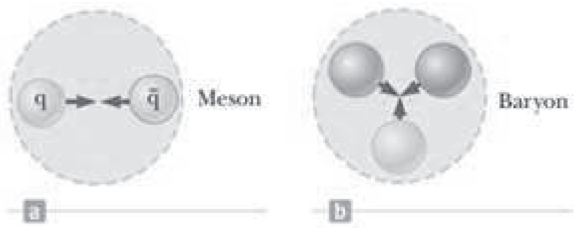

Fig 5.8 (a) A green quark is attracted to an anti-green quark. This forms a meson whose quark structure is (q). (b) Three quarks of different colours attract one another to form a baryon.

Although the nuclear force between two colourless hadrons is negligible at large separations, the net strong force between their constituent quarks is not exactly zero at small separations. This residual strong force is the nuclear force that binds protons and neutrons to form nuclei. It is similar to the force between two electric dipoles. Each dipole is electrically neutral. An electric field surrounds the dipoles, however, because of the separation of the positive and negative charges. As a result, an electric interaction occurs between the dipoles that is weaker than the force between single charges. We explored how this interaction results in the Van der Waals force between neutral molecules.

According to QCD, a more basic explanation of the nuclear force can be given in terms of quarks and gluons. Figure 5.9a shows the nuclear interaction between a neutron and a proton by means of Yukawa's pion, in this case a π^-. This drawing differs from the field particle is a π^0; there is no transfer of charge from one nucleon to the other. In Figure 5.9a, the charged pion carries charge from one nucleon to the other, so the nucleons change identities, with the proton becoming a neutron and the neutron becoming a proton.

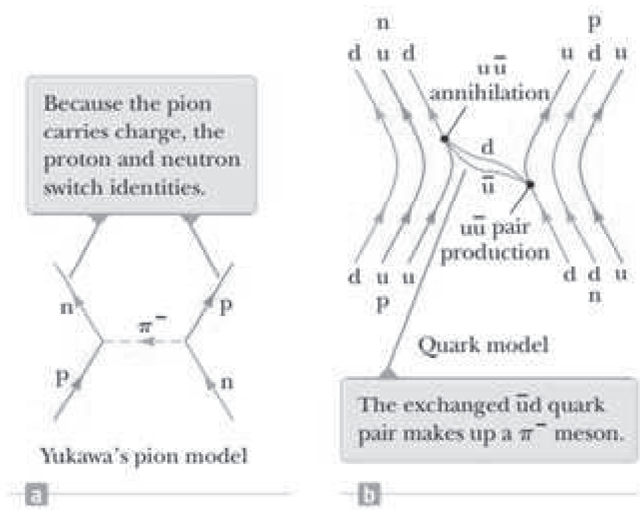

Fig 5.9 (a) A nuclear interaction between a proton and a neutron explained in terms of Yukawa's pion-exchange model (b) The same interaction, explained in terms of quarks and gluons

Although the nuclear force between two colourless hadrons is negligible at great distances, the net strong force between their constituent quarks is not zero at small distances. The nuclear force is the force that binds protons and neutrons together to create nuclei. It's similar to the force between two dipoles of electricity. Electrically, each dipole is inert. Because the positive and negative charges are separated, an electric field surrounds the dipoles. As a result, there is a lower electric contact between the dipoles than between single charges. The Van der Waals force between neutral molecules is the outcome of this interaction, as we've already seen.

According to QCD, a more basic explanation of the nuclear force can be given in terms of quarks and gluons. Figure 5.9a shows the nuclear interaction between a neutron and a proton by means of Yukawa's pion, in this case a π^-. This drawing differs from the field particle is a π^0; there is no transfer of charge from one nucleon to the other. The charged pion carries charge from one nucleon to the other, so the nucleons change identities, with the proton becoming a neutron and the neutron becoming a proton.

Let's look at the same interaction from the viewpoint of the quark model, shown in Figure 5.9b. In this Feynman diagram, the proton and neutron are represented by their quark constituents. Gluons are continuously emitted and absorbed by each quark in the neutron and proton. A gluon's energy can lead to the formation of quark–antiquark couples. In pair manufacturing, this process is identical to the generation of electron–positron pairs. When the neutron and proton approach to within 1 *fm* of each other, these gluons and quarks can be exchanged between the two nucleons, and such exchanges produce the nuclear force. Figure 8b depicts one possibility for the process shown in Figure 5.9a. A down quark in the neutron on the right emits a gluon. The energy of the gluon is then transformed to create a u pair. The *u* quark stays within the nucleon (which has now changed to a proton), and the recoiling d quark and the u antiquark are transmitted to the proton on the left side of the diagram. Here the u annihilates a u quark within the proton and the d is captured. The net effect is to change a u quark to a *d* quark, and the proton on the left has changed to a neutron.

As the d quark and antiquark in Figure 5.9b transfer between the nucleons, the *d* and exchange gluons with each other and can be considered to be bound to each other by means of the strong force. Looking at Table below, we see that this combination is a π^-, or Yukawa's field particle! Therefore, the quark model of interactions between nucleons is consistent with the pion-exchange model.

Table 5.5 *Quark Composition of Mesons*

		Antiquarks									
		$\bar{b}$		$\bar{c}$		$\bar{s}$		$\bar{d}$		$\bar{u}$	
Quarks	b	Υ	($\bar{b}b$)	B_c^-	($\bar{c}b$)	$\bar{B}_s^0$	($\bar{s}b$)	$\bar{B}_d^0$	($\bar{d}b$)	B^-	($\bar{u}b$)
	c	B_c^+	($\bar{b}c$)	J/Ψ	($\bar{c}c$)	D_s^+	($\bar{s}c$)	D^+	($\bar{d}c$)	D^0	($\bar{u}c$)
	s	B_s^0	($\bar{b}s$)	D_s^-	($\bar{c}s$)	η, η'	($\bar{s}s$)	$\bar{K}^0$	($\bar{d}s$)	K^-	($\bar{u}s$)
	d	B_d^0	($\bar{b}d$)	D^-	($\bar{c}d$)	K^0	($\bar{s}d$)	π^0, η, η'	($\bar{d}d$)	π^-	($\bar{u}d$)
	u	B^+	($\bar{b}u$)	$\bar{D}^0$	($\bar{c}u$)	K^+	($\bar{s}u$)	π^+	($\bar{d}u$)	π^0, η, η'	($\bar{u}u$)

Note: The top quark does not form mesons because it decays too quickly.

Check Your Progress

11. How many quarks is a baryon made up of?
12. What is quantum chromodynamics?

NOTES

5.10 ANSWERS TO CHECK YOUR PROGRESS QUESTIONS

1. In the mid-1930s, proton, electron, neutron, photon, neutrino and positron were considered to be the primary elements of matter.
2. A Feynman diagram is a qualitative representation of the relationship between time on the vertical axis and space on the horizontal axis.
3. Bosons are particles whose intrinsic angular momentum is a multiple of $\hbar$. Fermions are all particles with a spin that is only half integral.
4. Fundamental interactions can be defined as the fundamental forces that act between the primary particles that constitute all matter.
5. A quantum number is one of numerous integral or half-integral variables that uniquely identify the state of a physical system such as an atom, a nucleus, or a subatomic particle. Quantum numbers broadly refer to discrete (quantized) and conserved properties, such as energy, momentum, charge, baryon number, and lepton number.
6. The neutrino travels across space like a left-handed screw, but the antineutrino travels like a right-handed screw. Thus, the neutrino has a left-handed helicity, whereas the antineutrino has a right-handed helicity.
7. Charge conjugation is a mathematical process that turns a particle into an antiparticle by reversing the electric charge sign, for example every charged particle has an oppositely charged antimatter counterpart, or antiparticle, according to charge conjugation.
8. Physicists can distinguish between matter and antimatter with certainty because of the violation of CP symmetry.
9. Parity is a term that refers to the symmetry of the wave function used to describe the system.
10. In the absence of an electromagnetic field, one can envision a group of symmetry operators capable of transforming a neutron into a proton (or a proton into a neutron). The proton and neutron would therefore serve as the group's fundamental representations. The presence of such symmetry implies that something is constant in the presence of a strong interaction. This is referred to as isospin, and it is ½ for both the proton and neutron.
11. A baryon is made up of three quarks.
12. Quantum chromodynamics, or QCD, is the name given to the theory of how quarks interact with one another. In quantum chromodynamics, each quark is considered to have a colour charge, which is analogous to electric charge. The colour force is the strong interaction that exists between quarks.

5.11 SUMMARY

NOTES

- Hideki Yukawa, a Japanese physicist, proposed the first theory explaining the nature of the nuclear force in 1935, an accomplishment that earned him the 1949 Nobel Prize in Physics.
- The pion comes in three varieties, corresponding to three charge states: π^+, π^- and π^0. The π^+ and π^- particles (π^- is the antiparticle of π^+) each have a mass of 139.6 MeV/c^2, and the π^0 mass is 135.0 MeV/c^2. The two muons which exist are μ^- and its antiparticle μ^+. Pions and muons are very unstable particles.
- The interaction of two particles can be described graphically using a Feynman diagram. A Feynman diagram is a qualitative representation of the relationship between time on the vertical axis and space on the horizontal axis. It is qualitative in that the exact time and space values are irrelevant; however, the overall appearance of the graph provides a visual depiction of the process.
- The photon that transfers energy and momentum from one electron to the next is referred to as a virtual photon in the electron–electron interaction because it vanishes during the interaction without being observed.
- Fermi-Dirac statistics are applicable to particles constrained by the Pauli Exclusion Principle; fermions are particles that follow the Fermi-Dirac statistics. Fermions include leptons and quarks.
- In general, bosons operate as a medium for the transmission of forces between fermions; the photon, gluon, W, Z, and Higgs particles are all examples of bosons.
- The proton and neutron are the lightest members of the baryon class, whereas hyperons are the heaviest.
- The elementary particles are classified into two broad categories known as bosons and fermions. Bosons are particles whose intrinsic angular momentum is a multiple of ‘. Fermions are all particles with a spin that is only half integral.
- This category includes the proton and particles heavier than protons. Protons and neutrons are referred to as nucleons, while the others are referred to as hyperons. Each baryon has a corresponding antiparticle.
- Pions, kaon, and η- mesons are collectively referred to as mesons**.** Baryons and mesons are jointly called hadrons and are the particles of strong interaction.
- Fundamental interactions can be defined as the fundamental forces that act between the primary particles that constitute all matter. The fundamental interactions are classified as follows: (i) Strong interaction (ii) Electromagnetic interaction (iii) Weak interaction (iv) Gravitational interaction.
- The strong interaction is the force that holds nucleons together in the atomic nucleus (nuclear forces).

NOTES

- All interactions occur within a time range of approximately 10^{-23} s. Their degradation occurs over a period of around 10^{-10} s. Due to the length of time required for particles to respond to such an interaction, the force involved must be very weak in comparison to powerful nuclear forces.
- A quantum number is one of numerous integral or half-integral variables that uniquely identify the state of a physical system such as an atom, a nucleus, or a subatomic particle. Quantum numbers broadly refer to discrete (quantized) and conserved properties, such as energy, momentum, charge, baryon number, and lepton number.
- In intense contact, a quantity called hypercharge (Y) is conserved. The hypercharge of a particle is equal to the product of its strangeness and baryon numbers, $Y = S + B$. Because $B=0$ for mesons, hypercharge equals weirdness.
- A significant set of conservation laws is connected to parity (P), charge conjugation (C), and time reversal symmetries (T).
- Charge conjugation is a symmetry operation that replaces each particle in a system with its antiparticle. Charge parity (C) is conserved if the antisystem, or antimatter counterpart, exhibits the same physical phenomena.
- Antiparticles are defined as particles that are opposed to one another. They have the same mass and spin as one other, but they have the opposite charge. When they come into contact with one other, they annihilate each other by releasing photons into the environment.
- Particle-antiparticle annihilation would result in a significant amount of energy being released.
- Charge conjugation is a mathematical process that turns a particle into an antiparticle by reversing the electric charge sign, for example every charged particle has an oppositely charged antimatter counterpart, or antiparticle, according to charge conjugation.
- Quarks, the fundamental building blocks of K-mesons, exert a weak force on one another that causes this phenomenon. The weak force appears to act on a quantum mixture of two sorts of quarks, rather than on a pure quark state, as identified by the "flavour" or type of quark.
- Parity in time T denotes how a wave function behaves when t is replaced by $-t$. Time reversal is the symmetry operation that corresponds to the conservation of time parity. Time reversal symmetry indicates that the direction of time is irrelevant, and hence that the inverse of any process that is capable of occurring is also capable of occurring.
- In the absence of an electromagnetic field, one can envision a group of symmetry operators capable of transforming a neutron into a proton (or a proton into a neutron). The proton and neutron would therefore serve as the group's fundamental representations. The presence of such symmetry implies that something is constant in the presence of a strong interaction. This is referred to as isospin, and it is ½ for both the proton and neutron.

NOTES

- The quark model was proposed in 1964 by Murray Gell-Mann and G. Zweig. This theory is based on the notion that hadrons are constructed from a finite number of "fundamental" components known as quarks.
- Hadrons may be baryons or mesons. A baryon is made up of three quarks.
- A necessary component of the quark model with colour is that all observable meson and baryon states are "*colourless*", that is, either colour or anti-color combinations of mesons, or equal mixtures of R, B, and G for baryons.
- Hadrons are constructed entirely of six quarks and their antiquarks. The quarks' properties are inferred from the mesons and baryons' properties. To determine the masses of quarks from known hadron masses, we must first determine the strength of the interaction between quarks within the hadron.
- The nuclear contact between hadrons is mediated by massless field particles known as gluons, as previously explained. The nuclear force is a side effect of the strong force between quarks, as previously stated. The gluons are the strong force's mediators. The colour of a quark can vary when it emits or absorbs a gluon.
- Quantum chromodynamics, or QCD, is the name given to the theory of how quarks interact with one another, similar to *quantum electrodynamics* (the theory of the electrical interaction between light and matter). In quantum chromodynamics, each quark is considered to have a colour charge, which is analogous to electric charge.

5.12 KEY WORDS

- **Pion:** In particle physics, a pion is any of three subatomic particles: πp, πz, and π. Each pion consists of a quark and an antiquark and is therefore a meson. Pions are the lightest mesons and, more generally, the lightest hadrons.
- **Muon:** The muon is an elementary particle similar to the electron, with an electric charge of "1 e and a spin of 1/2, but with a much greater mass. It is classified as a lepton.
- **Quantum number:** It refers to the set of numbers used to describe the position and energy of the electron in an atom are called quantum numbers. There are four quantum numbers, namely, principal, azimuthal, magnetic and spin quantum numbers. The values of the conserved quantities of a quantum system are given by quantum numbers.
- **Antiparticle:** In particle physics, every type of particle is associated with an antiparticle with the same mass but with opposite physical charges.
- **Quarks:** A quark is a type of elementary particle and a fundamental constituent of matter. Quarks combine to form composite particles called hadrons, the most stable of which are protons and neutrons, the components of atomic nuclei.

5.13 SELF ASSESSMENT QUESTIONS AND EXERCISES

NOTES

Short Answer Questions

1. State the classification of elementary particles.
2. Write a short note on particles and anti-particles.
3. What do you mean by conservation of parity?
4. What is time reversal symmetry?
5. Write in brief about quark model.
6. What are multi-coloured quarks?

Long Answer Questions

1. Explain the types of fundamental interactions.
2. Describe the following:
   (a) Baryon Number
   (b) Lepton Number
   (c) Strangeness Number
   (d) Isospin and Isospin Number
3. Discuss the elementary idea of SU(2) and SU(3).
4. Explain Gell-Mann-Okubo mass formula.
5. Illustrate the compositions of hadrons according to the quark model.
6. Describe quantum chromodynamics.

5.14 FURTHER READINGS

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